EXPERIMENTAL PHYSICS

STONE

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BY

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PREFACE.

THIS book is the result of an experience of nearly ten years in teaching Experimental Physics to classes consisting of students who were preparing for college and of students who were not preparing for college.

Most of the experiments are quantitative, some are qualitative. Qualitative experiments serve to stimulate the interest of the student, and to prepare his mind for a better understanding of quantitative experiments. A beginner in Physics should know something about that which he is expected to measure before he attempts to measure it. This knowledge is readily acquired from qualitative experiments.

To show the aim of the work, I have put at the beginning of each experiment a concise statement, not of the result, but of the object of the experiment; and at the end of each experiment, questions for the purpose of helping the student unfold the result of the experiment from his record. The general results of the experiments are enforced by numerous examples, many of which have been drawn from Harvard Examination Papers. The experiments are often stepping-stones, each to the next.

The book contains only two or three experiments which require students to work in groups; for my experience

has shown that students get the greatest benefit from a laboratory course in Physics by working each for himself.

My purpose has been not only to teach the student something about Physics, but also, as Physics yields itself readily to this purpose, to teach him the importance of distinguishing between facts and inferences from these facts, to lead him to weigh facts carefully, and then to use his judgment impartially in drawing inferences. The teacher cannot use too much care in impressing upon the mind of the student the limitations which he must constantly put upon his statements and the danger of making generalizations from imperfect data. The student should not think that he is discovering laws of nature for himself.

I am under obligations to numerous authors and especially to my former teacher, Dr. E. H. Hall. Among many friends to whom my thanks are due are Professor G. A. Wentworth for his interest, encouragement, and suggestions, Professor J. A. Tufts for valuable assistance in reading both the manuscript and the proof sheets and for the great pains he has taken in looking after the English of the book, and Mr. Frank Rollins for reading the proof sheets and for valuable suggestions about the experiments.

I shall be grateful for any corrections or suggestions.

W. A. S.

EXETER, N. H., February, 1897.

THE LABORATORY AND THE APPARATUS.

THE laboratory should be a well-lighted room, provided with a sink, tables, and gas or gasoline, if experiments in heat are to be attempted. The room should also contain cases or cupboards, or a large closet, in which to store the apparatus when not in use. Some of the more common tools ought always to be at hand, such as a screw-driver, a hammer, a saw, a vise, a gimlet, a soldering-iron.

An intelligent carpenter can make many of the pieces of apparatus. In Boston, the L. E. Knott Apparatus Company, 16 Ashburton Place, are prepared to furnish the apparatus called for in this book. Some of the apparatus can doubtless be obtained from Walmsley, Fuller & Co., 134–136 Wabash Avenue, Chicago; Eimer & Amend, 205 Third Avenue, New York; Queen & Co., 1010 Chestnut Street, Philadelphia.

A sufficient supply of apparatus to provide for a division of twelve students working at a time will cost about $400. This estimate is based upon the fact that certain pieces of apparatus, such as balances and air-pumps, can be used in common by two or more students.

Some teachers may find useful the following list, which shows the experiments in this book that are similar to the exercises in the revised Harvard list or that are equivalent to them:

HARVARD EXERCISES.	EQUIVALENT EXPERIMENTS.	HARVARD EXERCISES.	EQUIVALENT EXPERIMENTS.
1.	1, 2, 3.	9.	108.
2.	7.	13.	109.
3.	13.	14.	110.
4.	9.	15.	110.
5.	6.	17.	88, 89.
6.	10.	18.	92.
7.	11, 12.	19.	93.

Harvard Exercises.	Equivalent Experiments.	Harvard Exercises.	Equivalent Experiments.
21.	95.	43.	49.
22.	96, 98.	44.	51.
23.	97.	45.	56.
24.	100.	46.	52.
25.	102.	47.	72.
26.	59, 60.	48.	78.
28.	61.	49.	77.
29.	62.	50.	129.
30.	63, 64, 65.	51.	139, 140, 141.
31.	66, 67, 68.	52.	142.
32.	25.	53.	138.
33.	28.	54.	143.
35.	106.	55.	144, 145.
36.	112.	56.	152, 153.
37.	113.	57.	154.
39.	36, 37, 38.	58.	147, 148, 149, 150.
40.	40.	59.	151.
41.	41.		

The experiments in the foregoing list are drawn largely from the Harvard Pamphlet, but in several cases with modifications. In general, apparatus has been recommended like that devised by Dr. E. H. Hall. For the discussion of the experiments valuable suggestions have been derived from Hall and Bergen's *Text-Book of Physics*, Hall's *Lessons in Physics*, and Worthington's *Physical Laboratory Practice*.

EXPERIMENTAL PHYSICS.

CHAPTER I.

MENSURATION, HYDROSTATICS, AND PNEUMATICS.

1. Purpose. The purpose of this course in physics is to lead the student to observe carefully, experiment intelligently, record accurately, judge impartially, and infer justly.

2. Directions for Note-Taking. In a note-book, which must be his constant companion in the laboratory, the student should keep a record of the experiments which he performs. This book must contain the *original* records of the work done. No records should be made on scraps of paper, to be copied later into the note-book. A blank book about 8¾ inches long by 7 inches wide, and containing about 250 pages of good unruled paper, suitable for pen or pencil, is recommended. The binding should be strong, with leather corners for the covers and a leather back. The first leaf should be left blank for the name and title, and the remaining pages numbered like the pages of a printed book, the even numbers on the left-hand page, the odd on the right.

The records of the measurements and of the observations should be put on the left-hand pages, while on the right-hand pages must be put computations and inferences. The notes, which may be written either with a black lead-pencil or in ink, should not be altered after they are made. An obvious error may be corrected by writing between the lines, but the original record should not be obscured in any way other than by drawing a line through the erroneous statement. At the top of the left-hand page must be placed the number and the object of the experiment with the date, and also the names of the pieces of apparatus used. The notes must not be crowded. It is a good plan to make illustrative diagrams and sketches. Not more than one experiment should be recorded on a page.

As long as the student does not depart from the general rules laid down, he is at liberty to follow any system of note-taking that may seem best to him. In developing his method of keeping notes, it is well for him to ask himself frequently, "Will my notes tell another person just what I have done?" Let the student make his notes concise, yet so clear that another in reading the record cannot fail to understand it.

3. Directions for Performing Experiments. Before beginning work, the student should read with care the directions which accompany the experiment. In these directions attention will be called to the precautions which should be taken in the proper performance of the experiment. A precaution once noted will be rarely mentioned again, but should be taken whenever applicable.

All measurements and other necessary data must be recorded. On the following pages, questions in connection with the experiments will be frequently asked. The questions must not be answered by a simple "yes" or "no," but by a declarative sentence. When the question is in parentheses, however, the student should not record the answer in his note-book, but should be able, when called upon, to give it orally.

4. Mensuration. With a meter stick, having inches on one side, measure the length, breadth, and thickness, as accurately as you can, of one of the table-tops in the laboratory. Get the dimensions in feet and fractions of a foot, also in meters and fractions of a meter. Record the measurements in your note-book, on page 2. After consulting your record, answer the following questions:

How many inches are there in each of the three dimensions? How many centimeters? How many millimeters?

In your records of numerical data and results, use decimal fractions only.

Experiment 1. *To find the volume of a solid of regular shape.*

Apparatus. A meter stick; a rectangular block of wood.

Directions. (*a*) Measure and record the length, breadth, and thickness of the block in inches and fractions of an inch, taking four measurements of the length, one along each of the four edges running in the direction of the length, four of the breadth, and four of the thickness. Find the average length, breadth, and thickness.

In making the measurements, place the meter stick on its narrow side (Fig. 1) to make the ends of the gradu-

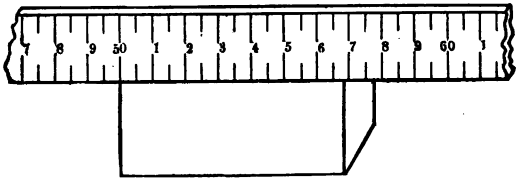


FIG. 1.

ations come close to the block; do not use the divisions at the ends of the stick, as the ends may be worn.

Find the product of the *numbers* that express the average length, breadth, and thickness. This product will be the number of cubic inches in the block.

(*b*) Using the same care as in (*a*), find the dimensions in centimeters and fractions of a centimeter.

From the average length, breadth, and thickness, find the number of cubic centimeters in the block.

NOTE. The block will be needed for the next two experiments.

Experiment 2. *To find the weight of a wooden block by means of a spring balance.*

Apparatus. The block of Exp. 1; a spring balance of 8-ounce capacity (Fig. 2); thread.

Directions. Taking care that the balance frame touches nothing, hang it by its ring from a hook or other suitable support. Place your head in such a position that the line of vision passes by the end of the pointer and is perpendicular to the face of the balance. Observe whether the pointer is opposite the line marked 0 (zero). If the

pointer is not opposite the zero line, note how much above or below it is. By means of a piece of fine thread hang the block on the hook of the balance, and observe the new position of the pointer. In computing the result, make a correction for the "zero error" of the balance, that is, the error arising if the pointer, when no weight is hung on the hook, is not exactly in front of the zero line. Strive to read carefully to the tenths of the smallest divisions.

FIG. 2.

Does the weight of the thread make any difference in the indications of the balance?

Making use of the result obtained in Exp. 1 (*a*), what do you find to be the weight in ounces of one cubic inch of the block?

Experiment 3. *To find the weight of a wooden block by means of a platform balance.*

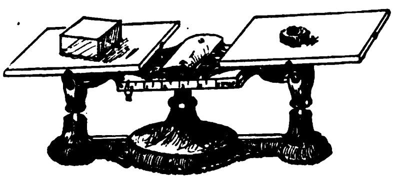


FIG. 3.

Apparatus. The block of Exp. 1; a platform balance; metric weights.

Directions. Put the rider on the zero notch of the balance scale. Wipe the pans dry and clean. Set the pans swinging, and add bits of paper till they swing evenly. On the left-hand pan lay the block; on the right-hand pan put weights (Fig. 3). Use the rider in making the final adjustments. Trust the indications of a swinging balance only. (Why?) Find how many grams and fractions of a gram the block weighs.

Making use of the results obtained in Exp. 1 (*b*), what do you find to be the weight in grams of one cubic centimeter of the block?

Experiment 4. *To find the volume of a solid of irregular shape.*

Apparatus. A 100^{cc} graduate (a cylindrical glass vessel marked off into cubic centimeters); a piece of lead.

Directions. Fill the graduate about half full of water and note the exact level of the water. Into the water put the piece of lead (Fig. 4). Be sure that it is entirely beneath the surface. (Why?) If air-bubbles cling to the lead, remove by shaking, but take care not to spill any water. Note the level at which the water now stands. As the surface of the water is highest at its edge where it meets the graduate, get the level by sighting along a horizontal line that just grazes the lowest part of the surface.

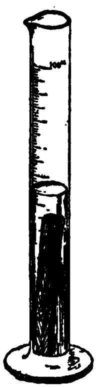


FIG. 4.

How many cubic centimeters of water are displaced? Is the volume of water displaced the same as the volume of the lead?

How many cubic centimeters does the lead contain?

5. Quantity; Unit; Numerical Value. In the experiments already performed we have made measurements of length, volume, and weight, and in our subsequent work, we shall often make measurements of other *magnitudes*, such as temperature, friction, and electrical resistance.

Measured magnitudes are called *quantities.* Every quantity is expressed by a phrase consisting of two parts: one of these is the name of a certain known quantity

which is taken as a standard of reference, and which is of the same kind as the quantity to be expressed; and the other is a number which shows how many times the standard is to be taken in order to make up the required quantity. The standard quantity is called the *unit*, and the number the *numerical value* of the quantity.

There are as many units as there are different kinds of quantities to be measured.

In this book the cubic inch is taken as the unit of volume and the ounce as the unit of weight for the English System, except for the measurement of large quantities, when the cubic foot and the pound are used; for the Metric System the cubic centimeter is taken as the unit of volume and the gram as the unit of weight.

The distance from one end of the Capitol at Washington to the other is just 751 feet. The phrase "751 feet" tells the number of units of a particular kind contained in the distance mentioned. In this case the *foot* is the *unit*, while 751 is the *numerical value* of the quantity.

QUESTIONS. What is the unit and what is the numerical value in each of the following expressions of quantity: 72 feet? 10 meters? 202 cubic centimeters? 8 grams? 7 ounces? 100 cubic feet? 10 inches?

DENSITY.

6. Density. The final result of Exp. 2 gave the weight in ounces of one cubic inch of the block, while that of Exp. 3 gave the weight in grams of one cubic centimeter of the block.

Definition. *By the density of a substance is meant the weight of one unit of volume of the substance.*

NOTE. Later in the course we shall limit the meaning of the word *weight* in this definition to that expressed by the word *mass*, and we shall show that a platform balance and not a spring balance should be used in our method of getting the density in the English System.

The final result of Exp. 2, then, gave the density of the block in the English System; the final result of Exp. 3, the density in the Metric System.

The student should consult Exps. 1, 2, and 3, and on the right-hand page of his note-book, the one opposite the last left-hand page that has been written on, give a brief account of the method of finding the density of a rectangular block of wood, which shall apply to either the English System or the Metric.

When the Metric System was planned, it was decided to take the density of water as unity, that is, to take the weight of a cubic centimeter of water as the unit of weight, and to call it the gram, hence

A cubic centimeter of water weighs a gram.

NOTE. Strictly speaking, however, in order that a cubic centimeter of water may weigh a gram, the water must be pure and it must have a certain temperature a few degrees above freezing; but for our purposes common water at any ordinary temperature will give results sufficiently accurate.

7. Precautions to be taken in Measurements and Computations. With a meter stick, such as you have used, suppose a rectangular block to have been measured in centimeters with the following results:

MEASUREMENTS OF LENGTH.	MEASUREMENTS OF BREADTH.	MEASUREMENTS OF THICKNESS.
cm.	cm.	cm.
7.30	6.40	3.12
7.32	6.40	3.10
7.32	6.41	3.11
7.33	6.42	3.14

It will be noticed that 7.30^{cm} in the record just given does not mean exactly the same thing as 7.3^{cm}. The former shows that the hundredths of a centimeter could be estimated, and that you have found that the length is more nearly 7.30^{cm} than 7.29^{cm} or 7.31^{cm}. The latter means that the hundredths of a centimeter were not taken into account. Consequently, unless you are careful to estimate the hundredths of centimeters, you must write not 7.30^{cm}, but 7.3^{cm}.

If we add each of the columns, and divide each sum by 4 to get the average length, breadth, and thickness, we have:

$$4\overline{)29.27} \qquad 4\overline{)25.63} \qquad 4\overline{)12.47}$$
$$7.32 \qquad\quad 6.41 \qquad\quad 3.12$$

Since the millimeter was the smallest division on the meter stick, and since we had to estimate the tenths of a millimeter, the second decimal place in each of the measurements is in doubt; hence the second decimal place in each of the averages is in doubt. Consequently, in each average, we have discarded as wholly doubtful all figures after the second place of decimals. It will be seen, however, by examining the averages, that the last figure retained has been increased by unity (one) whenever the first figure discarded is 5 or greater than 5. It is not only a waste of time to obtain and to record the average beyond the first doubtful figure, but is also inaccurate, since it is understood in physics that the observer claims as accurate all the figures in the record of a measurement except the last.

We have, then:

Average length	= 7.3**2**^{cm}.
Average breadth	= 6.4**1**^{cm}.
Average thickness	= 3.1**2**^{cm}.

For the sake of clearness the doubtful figures are printed in full-faced type.

Let us compute the volume of the block.

Volume of the block = 7.3**2** × 6.4**1** × 3.1**2**.

7.32	46.9
6.41	3.12
732	938
2928	469
4392	1407
46.9212	146.328

The final result is to be entered as 146^{cc}.

In the future it will be understood that the last figure recorded in the value of a measurement is in doubt.

Let the student turn back to his record of experiments performed, and, allowing the old computations to stand, reckon the results once more in accordance with the following rules:

1. *In all averages keep but one doubtful figure.* (If the figure following the doubtful one is 5 or greater than 5, increase the doubtful figure by unity.)

2. *After multiplying two numbers together, keep in the result as many figures of the product, counting from the left, as there are figures in the smaller factor.*

3. *After dividing one number by another, keep in the quotient, counting from the left, as many figures as there are in the smaller of the two.*

In subsequent experiments, these rules must always govern your measurements and computations.

SPECIFIC GRAVITY.

8. Specific Gravity. The object of the following experiment is to make clear to the student the meaning of the term *specific gravity.*

Experiment 5. *To find how many times as heavy as an equal volume of water a piece of lead is.*

Apparatus. A 100^{cc} graduate; a piece of lead that weighs more than 100^{g} (grams); a platform balance.

Directions. Find the weight in grams of a piece of dry lead. Find the volume of the lead in cubic centimeters (by method of Exp. 4).

With the data obtained, and knowing that *a cubic centimeter of water weighs a gram,* find how many times as heavy as an equal volume of water the lead is.

The number thus obtained is called the specific gravity of lead.

Definition. *The number obtained by dividing the weight of a substance by the weight of an equal volume of water is called the specific gravity of the substance.*

Note. For all work in this course use common water at any ordinary temperature.

9. Principle of Archimedes. The next two experiments have for their object the unfolding of the celebrated *Principle of Archimedes.*[1] The use of this principle will enable us to determine with ease the specific gravity of substances.

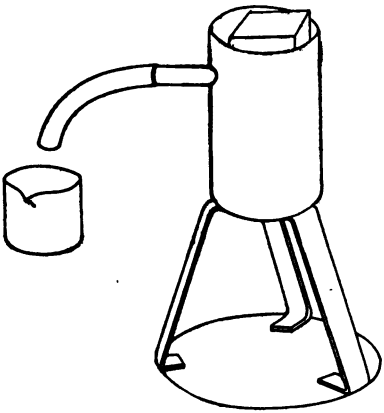


FIG. 5.

Experiment 6. *To find the relation between the weight of a body that will float in water and the weight of water it displaces.*

Apparatus. A 100^{cc} graduate; a 200^{cc} beaker; a copper vessel with a spout to which a rubber tube 10^{cm} long is attached; a wooden block whose corners are rounded, if necessary, to allow it to be put into the vessel; a platform balance.

Directions. Find the weight of the block in grams. Fill the vessel with water till some runs off through the tube attached to the spout. This tube should not be

[1] Archimēdes (287–212 B.C.), a famous mathematician of Syracuse in Sicily, was asked, so the story runs, by Hiero, king of Syracuse, to find whether a certain crown of the king's was of pure gold or of gold alloyed with silver. Archimedes asked for time to reflect on the problem. Soon afterwards, when in his bath, he noticed, what he had probably never carefully observed before, that his body was pressed upwards with a force which increased the more completely he was immersed in the water; to his subtle intellect this discovery suggested a way of solving the king's problem. By careful experiments he discovered that of equal masses of gold and silver the silver weighed less *in water* than the gold. On making the test, he found the crown to weigh less in water than an equal mass of gold, so he concluded that the crown was not of pure gold.

touched during the experiment. When water stops flowing, carefully put the block flatwise into the water (Fig. 5), and in the beaker catch all the water that flows out. Measure this water in the graduate.

How many cubic centimeters of water does the block displace?

How much does a cubic centimeter of water weigh?

How many grams of water does the block displace?

Is the weight of water displaced the same, or nearly the same, as the weight of the block?

Can you infer any relation between the weight of a body that will float and the weight of water it displaces?

QUESTION. If a wooden block weighs 100g, how many grams of water will it displace when floating in water?

Experiment 7. *To find how much less a body that will sink weighs in water than in air.*

Apparatus. A spring balance of 30-pound capacity; a heavy iron ball with a handle to which a string is attached (an iron safety-valve weight, weighing at least 12 pounds); a dipper; a pail.

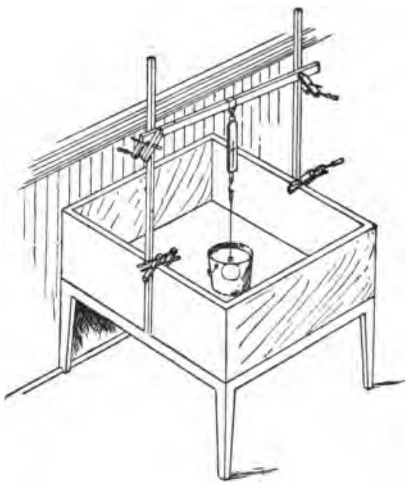


FIG. 6.

Directions. Perform the experiment at the sink. By its ring hang the balance to a stout support, and find the weight of the ball.

Then, taking care that the ball does not touch the pail, let it hang by the string, as in Fig. 6, from the hook of the balance so that it will be completely covered by the water in the pail in the sink. As it hangs immersed in the water, find its weight.

Remove the ball from the balance and hang the pail in its place. With the dipper fill the pail brimful of water, and get the weight. Now lower the ball into the pail until it is completely beneath the surface, holding it by the string till the water stops flowing over the edge of the pail; then carefully remove the ball, and note the weight of the pail and the water left.

How much less did the ball weigh in water than in air?

Did the ball displace its own volume of water?

What is the weight of the water displaced by the ball?

Can you infer any relation between the weight of water displaced and the apparent loss in weight of the ball?

The inferences from the results of Exps. 6 and 7 are together known as the Principle of Archimedes. Try to frame a concise statement that shall include both of your inferences.

QUESTION. If a cubic centimeter of iron weighs 7g, what will be its apparent weight if plunged under water?

10. Applications of the Principle of Archimedes. With one exception, the next six experiments involve the use of the Principle of Archimedes.

Experiment 8. *To find, without the use of a balance, the specific gravity of a piece of wood.*

Apparatus. The same as that used in Exp. 6, without the balance.

Directions. By the method of displacement of water (see Exp. 6) find the weight of the wood. Then press the wood down into the water until completely covered. Catch the water and measure it.

What do you find the volume of the wood to be in cubic centimeters? What, then, is the weight of a volume of water of the same size as the wood? What is the specific gravity of the wood? In what way has the Principle of Archimedes helped you in this experiment?

QUESTION. If the specific gravity of white wood is 0.5, how deep will a rectangular block of white wood sink in water?

Experiment 9. *To find, by submersion with a sinker, the specific gravity of a piece of wood.*

Apparatus. A rectangular block of wood; a platform balance standing on a wooden support; a piece of lead heavy enough to sink the block; a glass jar three-quarters full of water; a piece of thread.

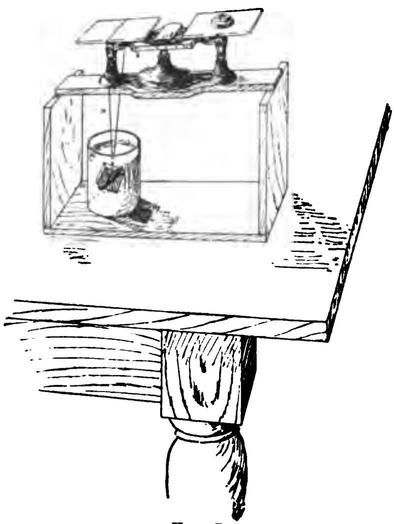


FIG. 7.

Directions. Weigh the block to 0.1^g. Over the left-hand pan of the balance, as shown in Fig. 7, loop the thread so that it will touch nothing but the pan and the water. Fasten the lead to the thread, and find how much the lead weighs when completely submerged in water without touching either the sides or the bottom of the jar. If air-bubbles are on the lead, brush them off.

Fasten the lead to the block, and, when entirely under water and touching neither the bottom nor the sides of the jar, weigh both to 0.1^g.

If the block were unattached to the sinker, and floating, by how many grams would the water buoy it up? (See your inference from Exp. 6.)

When the sinker is fastened to the wood, and both are completely submerged, by how many grams more does the water buoy up the block than it did before; that is, what is the difference between the weight of the sinker alone in water and the joint weight of sinker and block in water? What is the weight of a volume of water of the same size as that of the block? (In answering this question, consider carefully your answers to the two preceding ones.)

What is the specific gravity of the block?

NOTE. After drying the block, use it in the next experiment.

Experiment 10. *To find, by flotation, the specific gravity of a piece of wood.*

Apparatus. The block used in Exp. 9; a meter stick; a glass jar nearly full of water.

Directions. By taking measurements at each corner, get the average thickness of the block. Taking care that no air-bubbles cling to the under surface or to the sides of the block, gently lay it on the surface of water in the jar. If the sides of the block are not oiled, the water will creep up a little way, but if the block has been oiled, the water close to the block will be slightly depressed. [This phenomenon, either the creeping up of the water, or its

depression, belongs to a class included under the name *capillary action* (from the Latin *capillus*, hair), as the phenomenon was first observed in tubes of fine, hair-like bore.] With the eye on a level with the water, sight, through the sides of the jar, at the block, and see where the *general level* of the water would meet the block. Remove the block carefully. With a fine pointed lead-pencil, and guided by the water-line, make a dot at each corner where the water would have met the block had it not been for capillary action. Now measure from each of the dots to the lower surface of the block, and find the average depth to which the block sank.

The following is the course of reasoning by which the specific gravity may be found:

Imagine a block of water of the same size as the part of the wooden block under water.

What relation exists between the weight of this volume of water and the weight of the wooden block? (See your inference from Exp. 6.)

Imagine another block of water of the same size as the wooden block.

These two blocks of water, which you have imagined, have equal bases but unequal heights.

Is the weight of the thinner block the same part of the weight of the thicker that the height of the thinner block is of the height of the thicker?

What is the specific gravity of the wood?

In getting the specific gravity in this case, have you made direct use of the weight of the block and the weight of an equal volume of water?

Experiment 11. *To find, by the specific gravity bottle, the specific gravity of a liquid.*

Apparatus. A small bottle having a wide mouth (this is called the "specific gravity bottle"), with a glass stopple, and of 2-ounce or 3-ounce capacity; a platform balance; a piece of cloth or a towel with which to dry the bottle; water; kerosene.

Directions. Wipe the bottle dry both inside and out. Weigh the bottle together with the stopple, which should fit tightly. Weigh the bottle full of kerosene. When putting in the stopple, take care to exclude air-bubbles. The best way to do this is to fill the bottle brimful, then push the stopple into place. Weigh the bottle full of water.

What weight of kerosene did the bottle hold?

What weight of water did the bottle hold?

Was the volume of kerosene equal to that of the water?

What is the specific gravity of kerosene?

In getting the specific gravity in this case, have you made direct use of the weight of the kerosene and the weight of an equal volume of water?

Have you made use of the Principle of Archimedes?

Experiment 12. *To find, by means of buoyant action, the specific gravity of a liquid.*

Apparatus. A jar three-quarters full of kerosene; a jar three-quarters full of water; a platform balance with support; a piece of lead or iron weighing 100^g or more; a piece of thread.

Directions. Weigh the piece of lead. With the thread suspend the lead from the balance pan, and weigh in kerosene. Take the lead from the kerosene, wipe dry, and find its weight in water.

What weight of water did the lead displace? (See your inference from Exp. 7.) What weight of kerosene did the lead displace? Was the volume of kerosene displaced by the lead the same as the volume of water?

What is the specific gravity of kerosene?

Have you made use of the Principle of Archimedes?

Now that you have performed the experiment, what, should you say, is the meaning of the term "buoyant[1] action"?

Experiment 13. *To find the specific gravity of a solid that will sink in water.*

Apparatus. A glass bottle without stopple (the glass bottle used in Exp. 11 will answer); a platform balance on a support; a piece of thread; a jar of water.

Directions. Weigh the bottle *empty*. Also weigh the bottle in water, *after filling it with water*. See that every part of the bottle is beneath the surface of the water, and that there are no air-bubbles in the bottle.

What is the weight of a volume of water of the same size as the volume of the glass?

What is the specific gravity of the glass?

Had the glass forming the bottle been in a solid lump, would the result have been different?

Have you made use of the Principle of Archimedes?

11. Agreement between Specific Gravity and Density in the Metric System. If the student compares the result of either Exps. 8 or 9 with that of Exp. 2 and that of Exp. 3, the specific gravity of wood with its

[1] When a word or a term occurs the meaning of which is not perfectly clear, consult a good dictionary.

density in the English System and in the Metric, he cannot fail to notice that the specific gravity is widely different from the numerical value of the density in the English System, while it agrees closely with the numerical value of the density in the Metric. This agreement does not come by chance, but is the result of the relation which, in the Metric System of weights and measures, exists between the unit of volume and the unit of weight.

For example, to get the specific gravity of a piece of wood, we divide its weight by the weight of an equal volume of water; on the other hand, to get the density of the piece of wood, we divide its weight by the numerical value of its volume. In the Metric System one cubic centimeter of water weighs one gram; so the numerical value of the volume of a body is equal to the numerical value of the weight of a like volume of water; hence, the quotients resulting from the divisions must be equal in numerical value.

EXAMPLES.

1. A piece of wood weighs 75^g, and its density is 0.5^g per cubic centimeter. What is its volume?

2. A body weighs 100^g in air, and 75^g in water. What is the specific gravity of the body?

3. A body lighter than water weighs 100^g in air. A sinker, weighing 50^g in water, is attached to the body, and their combined weight in water is 25^g. What is the specific gravity of the body?

4. What is the specific gravity of water?

5. A cubic decimeter (a cube whose edges are 10^{cm}) of wood sinks to a depth of 6^{cm} in water. Find the specific gravity of the wood.

6. A specific gravity bottle weighs, when empty, 1000^g; when filled with kerosene, 1800^g; when filled with water, 2000^g. Find the specific gravity of kerosene.

7. Weight of a sinker in air, 50^g; in kerosene, 42.1^g; in water, 40^g. Find the specific gravity of kerosene.

8. A piece of wood of irregular shape floats with $\frac{b}{a}$ of its volume under water. What is its specific gravity?

Solution. If we let 1 represent the weight of a volume of water equal to that of the wood, the weight of the wood will (by Principle of Archimedes, Exp. 6) be $\frac{b}{a} \times 1$, or $\frac{b}{a}$. Hence sp. gr. $= \frac{\frac{b}{a}}{1} = \frac{b}{a}$.

9. An iceberg floats with one-ninth of its volume above the water. Find the specific gravity of the iceberg.

10. Let us suppose that Hiero's crown (see Art. 9) weighed 10 ounces, and that its specific gravity was 15. If the specific gravity of gold is 19.3, and the specific gravity of silver is 10.5, find the weight of silver in the crown.

Solution. Let x denote the weight of silver in the crown.
Then $10 - x$ will " " " " gold " " "

Weight of water whose volume equals that of silver $= \frac{x}{10.5}$.

" " " " " " " " gold $= \frac{10-x}{19.3}$.

By the definition of specific gravity, we have $\dfrac{10}{\frac{x}{10.5} + \frac{10-x}{19.3}} = 15.$

Clearing of fractions, taking out common factors, and combining, we find $8.8x = 30.1$, whence $x = 3.42$ ounces.

11. A diamond ring weighs 65 grains in air, and 60 in water. Find the weight of the diamond, the specific gravity of gold being 17.5, and that of the diamond 3.5.

PNEUMATICS.

12. Air. The earth on which we live is surrounded by a gas which we call air. Although we cannot see it, its existence is proved to us by our being able to feel it. When we are riding fast we feel the air. The whole mass of this air is called the *atmosphere*.

We shall now study in the next three experiments some of the properties of air which come under the head of *Pneumatics*.

Experiment 14. *To find whether air has weight.*

Apparatus. An air-pump; a piece of "pressure" tube with a pointed glass tube in one end attached to the exhaust nozzle; a 2-quart glass bottle with a perforated rubber stopple, having a glass tube thrust through with a piece of pressure tube 10^{cm} long attached; a small brass clamp; a platform balance; vaseline.

Directions. Into the mouth of the bottle insert the stopple slightly smeared with vaseline to make the bottle air-tight. Over the tubing slip the clamp so that it fits loosely.

Weigh the bottle together with stopple, tubes, and clamp. The result is the weight of the bottle filled with air. Lay the bottle on the table close to the air-pump. Push the end of the rubber tube over the pointed glass tube attached to the pump. Without hurrying, make 20 strokes of the pump (a *stroke* is one complete up-and-down movement of the handle). Some of the air has now been pumped out. Screw the clamp tightly on the rubber tube attached to the bottle. Remove the bottle and weigh.

If there is a difference between the two weights, how do you account for it?

Experiment 15. *To find whether the atmosphere presses equally in all directions.*

Apparatus. An air-pump; a thick-walled "thistle-tube" whose mouth, about 2.5^{cm} in diameter, has a piece of thin sheet rubber, like that which dentists use, stretched across, and firmly fastened by a piece of strong thread wound round the tube just back of the lip, as shown in Fig. 8 (the piece of sheet rubber thus arranged is called a *diaphragm*); a piece of pressure tube 30^{cm} long.

Directions. By means of the pressure tube connect the stem of the thistle-tube with the air-pump. Make one or two strokes of the air-pump. *Take care not to burst the diaphragm.* The diaphragm will be forced into a deep cup-shape by the pressure of the atmosphere. To prevent air from leaking into the thistle-tube from the pump, pinch the tube tightly. Watching the curvature of the diaphragm, turn the mouth of the tube up, down, and in many directions.

As you turn the mouth of the tube in different directions, does the shape of the diaphragm change?

What inference can you draw?

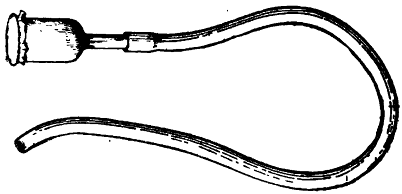


FIG. 8.

Pneumatics is that branch of physics which treats of the mechanical properties of air, as weight and pressure.

Experiment 16. *To find whether the pressure of the atmosphere upon a given surface depends on the shape of the passage by which it reaches the surface.*

Apparatus. A small thistle-tube with a rubber diaphragm across the mouth, about 1.5^{cm} in diameter, and a piece of rubber tube 50^{cm} long attached to the stem.

Directions. With one side of the diaphragm the air communicates directly, but with the other indirectly through the tube. If, without compressing the tube, you bend it into different shapes, you will be able, by

watching the diaphragm, to decide whether there is any change of pressure on the "given surface," the side of the diaphragm in the thistle-tube.

What inference can you draw from this experiment?

LIQUID PRESSURE.

13. Liquid Pressure. We shall now study the laws which govern pressure in liquids.

Experiment 17. *To find whether the pressure increases with the depth below the surface.*

Apparatus. A pail three-quarters full of water; a pressure-gauge.

Directions for Making the Pressure-Gauge. Beneath the surface of water in the pail dip one end of a piece of thermometer tube whose length is about 15^{cm}. When

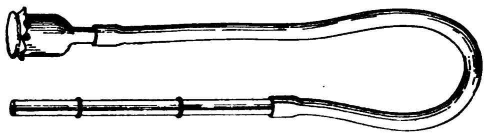


FIG. 9.

the end is about 1^{cm} below the surface, cover it with the finger and remove the tube. The little thread of water about 1^{cm} long thus caught in the tube is called an index. Turn the tube into a horizontal position, and remove the finger from the end. By gentle shaking, get the index placed at about one-fourth the length of the tube from one end; over this end, which should have been heated

and drawn out, carefully slip the rubber tube attached to the thistle-tube of Exp. 16, and push it on till it covers about 1^{cm} of the glass tube. Take care not to kink, or bend sharply, or compress the rubber tube, lest you force the index out. Make a rubber ring by cutting a thin slice from a rubber tube. Push the glass tube into this ring. The gauge is now completed, as shown in Fig. 9. In using it, *be sure the tube containing the index is horizontal.*

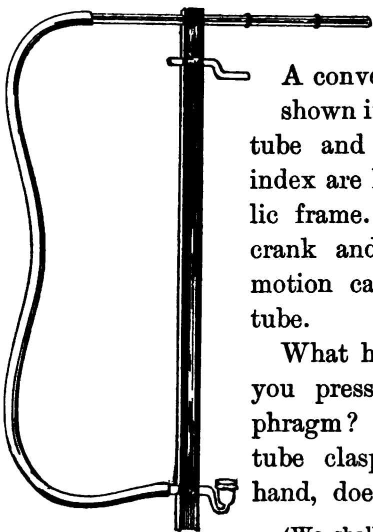


FIG. 10.

A convenient form of the gauge is shown in Fig. 10, where the thistle-tube and the tube containing the index are held in position by a metallic frame. By means of the little crank and a rubber belt, a rotary motion can be given to the thistle-tube.

What happens to the index when you press lightly against the diaphragm? If you hold the thistle-tube clasped in the palm of your hand, does the index move?

(We shall study this question more carefully in the chapter on Heat.)

Directions. Beside the pail upon the table lay the glass tube of the gauge; along this tube push the little rubber ring (the "marker") till it is over one end of the index. With one hand steady the glass tube, with the other, using great care *not to kink, or bend sharply, or compress* the rubber tube, lower the thistle-tube into the pail (Fig. 11), which should contain water that has stood

in the room for several hours, to ensure that its temperature shall be nearly the same as that of the room. As the thistle-tube goes deeper and deeper below the surface, watch the index.

How does the index act? What inference can you draw from this action? With what are the thistle-tube and the rubber tube filled? By what means is the pressure sent from the diaphragm to the index?

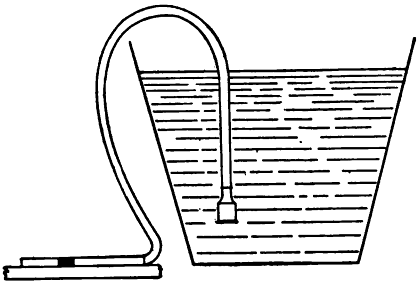


FIG. 11.

Experiment 18. *To find whether the pressure at a given point in a liquid is the same in all directions.*

Apparatus. The same as in Exp. 17.

Directions. On the inside of the pail, a little way below the surface of the water, make a dot. Keeping its center on a level with this dot and at a given distance from the dot, turn the diaphragm so that it will face in various directions, downwards, horizontally, and obliquely.

What do you infer from the action of the index?

Compare your inference with that of Exp. 15.

When, as in this experiment, the tube of the gauge is bent into different curves, what reason have you for thinking that the pressure transmitted by the air in the tube does not change in passing round the curves? (See your inference from Exp. 16.)

Experiment 19. *To find whether the pressure at all points, in a horizontal plane passing through the liquid, is equally great.*

Apparatus. The pressure-gauge; two student-lamp chimneys; a retort-stand with two clamps each large enough to hold a chimney; two good cork stopples to fit the smaller end of each chimney; a pail of water; some cement made by melting together equal parts of beeswax and rosin.

PART 1. Where it is possible to pass along a straight line from one point to any other in the plane.

Directions. Dip the stopples into the melted cement and stop the smaller end of each chimney air-tight. Fill

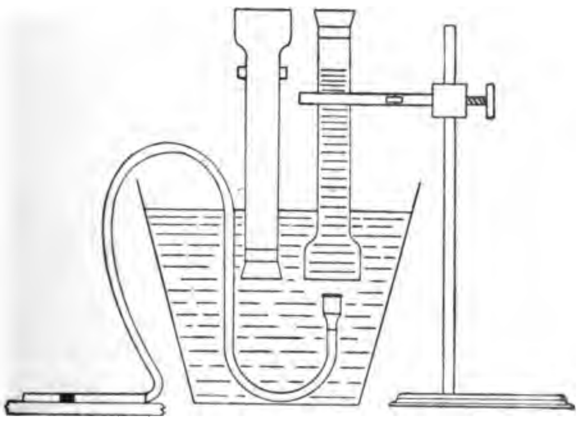


FIG. 12.

one of the chimneys with water. Placing the hand or a piece of wet paper over the open end, invert and lower the chimney into the pail of water. When the covered end is below the surface of the water, remove the hand or paper. With its lower end dipping about 6^{cm} below the

surface of the water, clamp the chimney in a vertical position. (Why the water remains in the chimney you will learn from Exp. 20.) With its closed end downwards, and about 6^{cm} below the surface of water, clamp the other chimney in a vertical position. Push the diaphragm of the gauge about 8^{cm} below the surface of the water. Sometimes bringing the diaphragm under the chimney filled with water, as shown in Fig. 12, sometimes under the chimney whose closed end is in the water, and at other times under neither, carefully move the diaphragm about in the *same horizontal plane*.

Do you observe any change of pressure in going from one point to another in the same horizontal plane?

What is your inference?

As you moved the diaphragm from place to place, the depth of water over it was sometimes greater and sometimes less. How many different depths of water were there?

PART 2. Where it is impossible to pass along a straight line from one point to another in the plane.

Directions. Push the chimney filled with water down till its open end is about 10^{cm} below the level of the water in the pail. Clamp the chimney. Push the diaphragm up inside the chimney about 2^{cm}. With the marker note the position of the index. Then, with the tube bent in as nearly the same shape as before, put the diaphragm in the water outside, but on a level with its former position in the chimney. Observe the position of the index. Great care is necessary to avoid compressing the rubber tube.

In this part of the experiment, the barrier that has separated one portion of the horizontal plane from the other has been the walls of the chimney.

What inference can you draw?

EXAMPLES.

1. A rectangular vessel, whose interior dimensions are: width 10^{cm}, length 15^{cm}, and height 20^{cm}, is filled with water.

a. What is the weight of water in the vessel?

b. What is the weight of water resting on each square centimeter of the base?

c. What is the pressure upon a horizontal square centimeter at the depth of 2^{cm}? At a depth of 10^{cm}? At a depth of 15^{cm}?

2. Suppose the vessel of the preceding example to be closed by a flat cover with a hole through its center, into which fits an open tube of $1^{sq\,cm}$ cross-section, and also suppose this tube to extend upwards 30^{cm} above the top of the vessel. Suppose both vessel and tube filled with water.

a. What is the total weight of water in vessel and tube?

b. What is the pressure upon that square centimeter of the base of the vessel which lies exactly beneath the tube?

c. Is the pressure upon any other square centimeter of the base of the vessel greater or less than this?

d. What is the pressure upon that square centimeter which, at the top of the vessel, lies exactly beneath the tube?

e. Is the pressure against each square centimeter of the cover greater or less than this?

f. What is the total pressure of the water against the cover?

g. What is the total pressure of the water against the base?

h. Subtract the total pressure against the cover from the total pressure against the base, and compare the result with the weight of all the water in the vessel and tube.

i. What is the total pressure upon $1^{sq\,cm}$ of the vertical side of the vessel, the center of the square being at a depth of 5^{cm} beneath the cover? At a depth of 10^{cm}? At a depth of 15^{cm}?

j. What is the total pressure upon one of the narrow vertical sides of the vessel? Upon one of the broad vertical sides?

k. What is the total pressure upon the base, cover, and sides?

l. Suppose, by means of a piston, for example, a pressure of 50^g is brought to bear upon the top of the water in the tube. What will now be the pressure upon that square centimeter which lies at the top of the vessel, just beneath the tube?

m. How much will the total pressure against the base of the vessel be increased by the action of the added pressure?

n. How much will the total pressure against the base, cover, and sides be increased by the action of the added pressure?

ATMOSPHERIC PRESSURE.

14. Pressure of the Atmosphere. In Exp. 19 you found that the lamp-chimney remained full of water after it was inverted in the pail of water. In the following experiment we shall study the cause of this.

Experiment 20. *To find whether it was the pressure of the atmosphere that kept the water in the chimney.*

Apparatus. In place of the chimney, a glass tube about 100^{cm} long and 0.5^{cm} in diameter; a good cork stopple to fit the tube; a pail of water.

Directions. With the stopple close one end of the tube, then fill it with water. Taking care not to shut in an air-bubble, press the finger firmly against the open end. Place the end covered with the finger under the surface of the water in the pail and then remove the finger.

Does the water in the tube fall?

When you remove the stopple, what is the result?

Before the stopple was removed, did the atmosphere get to the water in the tube to press it down? Did the atmosphere get to the water in the tube to hold it up?

If the atmosphere did not get at the water in the tube directly to hold it up, by what indirect means was the pressure of the atmosphere transmitted to the water in the tube? What inference can you draw?

15. Precautions to be taken in Using Mercury. It would be interesting to find how tall a column of water the pressure of the atmosphere would support; unfortunately, however, a tube of sufficient length for the water would be hard to manage; so we make use of a short tube and mercury, a liquid much heavier than water. Mercury, or quicksilver, as it is often called, has a specific gravity of 13.6. We shall use mercury in several experiments. The vapor of mercury is poisonous, hence do not heat mercury in the room. Before beginning an experiment in which mercury is used, remove rings from the fingers, as it badly stains gold. In performing experiments with mercury, place all the apparatus in a shallow pan to catch any mercury accidentally spilt.

Experiment 21. *To find how tall a column of mercury the pressure of the atmosphere will support.*

NOTE. This is often called the "Torricellian Experiment," in honor of Torricelli (pronounced *tor-re-chel'lee*), an Italian who performed the experiment in 1643.

Apparatus. A piece of thick glass tube, whose bore is about 0.6^{cm} in diameter, about 80^{cm} long, closed at one end; a retort-stand with clamp; an iron pan; a small mortar; a small funnel; a piece of iron wire; a piece of cloth; mercury; a meter stick.

Directions. Make a soft pad of the cloth and lay it in the pan. On the pad rest the closed end of the tube, and clamp in a vertical position. By the aid of the funnel half fill the tube with mercury; then, by twisting the

wire in the tube, remove the air-bubbles which adhere to the sides. Add a little more mercury and again remove any air-bubbles that you may see. When the tube is completely filled with mercury and contains no air-bubbles, have ready in the pan the mortar filled with mercury to a depth of about 3^{cm}. Placing the finger firmly over the open end, grasp the tube near the top with the right hand. With the left hand unclamp the tube, grasp it near the bottom, and inverting the tube, place the end in the mortar below the level of the mercury. Taking care that no air enters, remove the finger and again clamp the tube in a vertical position, as in Fig. 13.

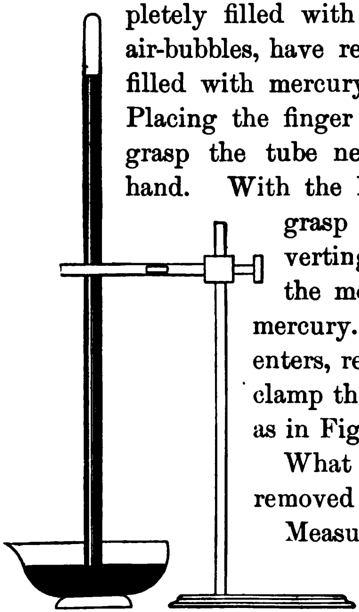


Fig. 13.

What happens when the finger is removed?

Measure the distance from the level of the mercury in the mortar to the top of the column of mercury in the tube.

What relation is there between the pressure at a point on the surface of the mercury in the mortar and the pressure at a point on the same level in the mercury in the tube? (See your inference from Exp. 19, Part 2.)

Experiment 22. *To find what weight of mercury in the tube the atmospheric pressure holds up.*

Apparatus. The same as that used in Exp. 21; a glass beaker; a platform balance.

Directions. Using the same care as in Exp. 21, fill the tube and invert it in the mortar partly filled with mercury. When the top of the mercury column has become stationary, find its height as in the last experiment. Over the lower end of the tube put the finger loosely, and gently raise the tube till its lower end is just beneath the level of the mercury. Now press the finger firmly against the end of the tube, lift the tube out, and incline it in a nearly horizontal position. By loosening the finger admit the air, a few bubbles at a time, and allow the mercury to run very gently into the beaker. Do not spill any of the mercury. Weigh the beaker with the mercury, and also weigh it empty.

What weight of mercury did the air hold up?

In taking the tube from the mortar, why did you raise it till its lower end was just beneath the surface of the mercury before pressing firmly with the finger?

Making use of the height and weight of the mercury column, and of the fact that the specific gravity of mercury is 13.6, answer the following questions:

What was the volume in cubic centimeters of the mercury in the tube? What was the area in square centimeters of the cross-section of the tube? What was the pressure in grams of the atmosphere upon an area of $1^{sq\ cm}$? What would be the length in centimeters of a column of water supported by the atmospheric pressure?

16. The Barometer. The *barometer* is an instrument for indicating the changes in the pressure of the atmosphere. It consists of a cistern filled with clean mercury into which dips an upright tube of glass containing mercury and closed at the upper end; the arrangement is

like that of Exp. 21. Alongside the tube stands a scale to measure the height of the mercury column. By watching the mercury column it has been found to vary in length not only from day to day, but frequently also many times a day. This shows that the pressure of the atmosphere is sometimes greater, sometimes less. The pressure of the atmosphere is expressed by the length of the column of mercury which it supports. Thus, if the column at one time is 72^{cm} tall, at another 80^{cm}, we say that on the first occasion the pressure of the atmosphere was 72^{cm}, on the other 80^{cm}. The *length* of the mercury column is called the *height* of the barometer. The average height of the barometer at the sea level is about 76^{cm}. In climbing a mountain the atmospheric pressure becomes less and less the higher you go, so if you should carry a barometer from the base to the top of a mountain, the height of the column of mercury would decrease. The space above the mercury in a barometer is *nearly* a perfect vacuum; in honor of Torricelli, who first observed this, it is called a Torricellian vacuum. The presence of the vapor of mercury in this space prevents it from being a perfect vacuum.

The term "barometric pressure" is often used in place of the term "atmospheric pressure."

17. Balancing Columns. After a liquid has been poured into connecting tubes (Fig. 14), the column of liquid in one branch is said to balance the column of liquid in the other. In order that the results obtained in experiments with balancing columns may be uninfluenced by the *capillary action* (see page 17), the branches of the connecting tubes must be sufficiently wide to allow

the center of the surface of the liquid in each column to lie flat.

Experiment 23. *To find whether two balancing columns of water contained in connecting tubes of unequal cross-section are of equal height.*

Apparatus. Two connecting tubes of glass of unequal cross-section, as shown in Fig. 14, one about 0.5^{cm} in diameter, the other about 1^{cm}.

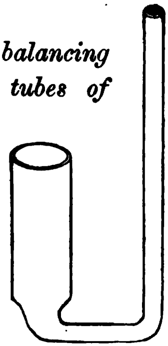


FIG. 14.

Directions. Pour water into the larger tube.

How does the height of the water in the larger tube compare with the height of the water in the smaller?

If the two columns of water are of the same height, what supports the extra weight of the column in the larger tube?

Could the walls of the tube, where they narrow, support this weight?

Experiment 24. *To find the specific gravity of a liquid by balancing columns.*

Apparatus. A support consisting of a square base with an upright rod about 100^{cm} long, to which is fastened a meter stick; a piece of rubber tube; two glass tubes, each about 100^{cm} long and about 0.6^{cm} in diameter; a funnel; a beaker; two rubber bands.

Directions. To a distance of about 1^{cm} over an end of each of the glass tubes slip an end of the rubber tube. Place the two glass tubes parallel to each other so that they with the rubber tube look like a very tall letter U (we shall call this a U-tube). With the two rubber bands, as shown in Fig. 15, one near the top, the other about

half way down, firmly fasten the U-tube to the upright of the support. Have the rubber tube so bent as to insure free communication between the two branches. In order to read easily the level of the liquids, have a branch of the U-tube at either edge of the meter stick.

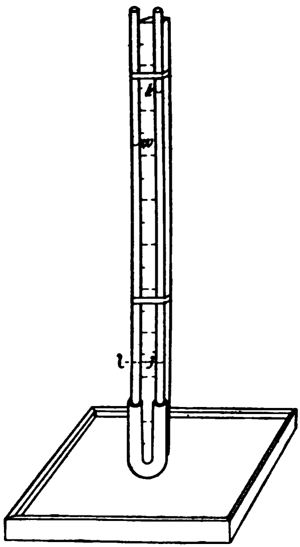


FIG. 15.

Into one branch pour water through the funnel till it about half fills both. By drawing the fingers pressed together along the rubber tube, drive out any hidden air-bubbles. Into one of the branches, very slowly at first, pour kerosene. Notice the well-defined boundary between the kerosene and the water. Continue pouring kerosene till the top, *k*, of the column of kerosene is nearly on a level with the top of the meter stick. Be sure, however, that the boundary, *j*, between the kerosene and the water does not sink out of sight into the rubber tube.

How does the pressure at the boundary of the two liquids compare with the pressure at the same level in the water in the other tube? (See your inference from Exp. 19, Part 2.)

How does the weight of the kerosene column, *kj*, compare with the weight of the water column, *lw*, which stands above the horizontal plane passing through the boundary of the kerosene and the water?

What is the length of this column of water?

What is the length of the kerosene column?

How does the weight of a certain length of the kerosene column compare with the weight of an equal length of the water column?

What is the specific gravity of the kerosene?

QUESTION. When the two branches of a U-tube have the same area of cross-section, suppose a column of water 80^{cm} tall balances a column of kerosene 100^{cm} tall; how tall a column of kerosene would a column of water 80^{cm} tall balance, if the branches of the U-tube were of unequal area of cross-section?

18. Inverted U-Tube. Even when the liquid will mix with water, the method of balancing columns can be used, but in a modified form, as described in the following experiment.

Experiment 25. *To find the specific gravity of a liquid by means of the inverted U-tube.*

Apparatus. The support and the glass tubes of Exp. 24; two small tumblers; a lead three-way tube; a pinch-cock; rubber tubing; rubber bands.

Directions. By means of short pieces of rubber tube couple a glass tube to each of the parallel arms of the three-way tube. Over the remaining arm slip a somewhat longer piece of rubber tube, which is clasped loosely by the pinch-cock. Fill one tumbler with water, the other with kerosene, and place them side by side upon the base of the support, which is not shown in Fig. 16. To this support fasten, in an inverted position, with rubber bands, the U-tube that you have just made, with one of the glass tubes dipping into the water, the other into the kerosene.

Record the height to which capillary attraction raises the liquids in each tube. Then by means of the mouth draw out some of the air through the rubber tube till the liquids rise to a considerable height in each tube, and, finally, while the liquids are raised in this way, clamp the rubber tube with the pinch-cock.

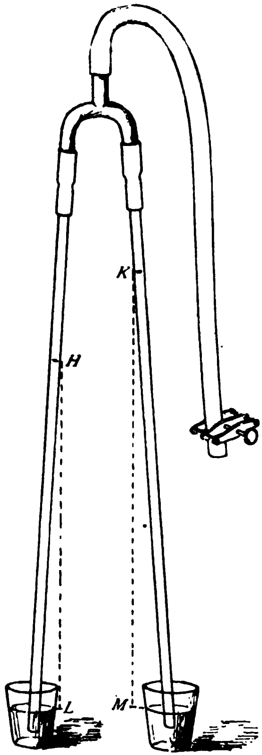


FIG. 16.

Record the height of the top of each column of liquid above the liquid in the tumbler. In order to find the heights to which the liquids actually rose by reason of removing some of the air from the tubes, subtract from each of the recorded heights the amount of elevation due to capillary attraction at first observed in each tube. When thus corrected, what is the true height of the water column, and of the kerosene column?

How does the weight of a certain length of the kerosene column compare with the weight of an equal length of the water column? What is the specific gravity of the kerosene?

SPECIFIC GRAVITY OF AIR.

19. Specific Gravity of Air. We have become acquainted with methods for finding the specific gravity

of solids and of liquids. In the following experiment we shall get the specific gravity of air by a method that will serve to illustrate roughly the way in which the specific gravity of gases may be found.

Experiment 26. *To find the specific gravity of air.*

Apparatus. An air-pump of simple construction (see Fig. 19); a dry 2-quart bottle with a perforated rubber stopple, through which is thrust a piece of glass tube long enough almost to touch the bottom of the bottle and to project about 2^{cm} above the stopple; a piece of pressure tube about 30^{cm} long; a brass clamp; a 250^{cc} graduate; a glass jar filled with water; a platform balance.

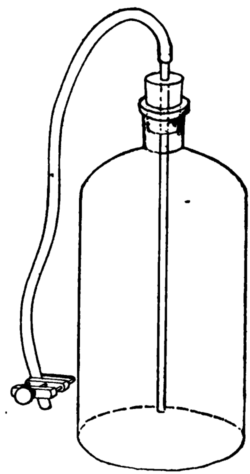


FIG. 17.

Directions. Into the mouth of the bottle insert the stopple slightly smeared with vaseline to make the bottle air-tight. Over the end of the glass tube outside of the bottle slip the pressure tube and put the clamp, as shown in Fig. 17, loosely over the tube.

Weigh the bottle carefully, together with tubes, stopple, and clamp. Lay the bottle on its side close to the air-pump. Over the exhaust nozzle of the pump slip the end of the pressure tube. Make strokes of the pump to the number of 20 or 40, or until the air as it escapes from the pump makes a short, faint hiss. Close to the end of the pressure tube next to the air-pump make the clamp fast. Detach the bottle and weigh it carefully.

What weight of air has been removed? (If less than

0.7^g has been removed, pump out some more air and weigh again.)

Near the edge of the table place the bottle and the jar of water. To a considerable depth beneath the surface of the water plunge the end of the pressure tube and loosen the clamp. As the water runs in, hold the bottle upright at the edge of the table close beside the jar. By raising or lowering the bottle, keep the surface of water in it on the same level with the surface of water in the jar.

When the clamp was loosened, what made the water rush into the bottle? When the water stops flowing into the bottle, what is the relation between the pressure of the atmosphere and the pressure of the air in the bottle? What is the reason for your answer?

Clamp the end of the pressure tube and loosen the stopple. Then raise the tube into a vertical position, loosen the clamp, and allow the water in the tube to run into the bottle. (Why?) Now find how many cubic centimeters of water there are in the bottle by measuring this water with the graduate.

How many cubic centimeters of air at atmospheric pressure have been removed from the bottle?

How many grams does this air weigh? (See answer to question as to weight of air removed.)

What is the weight of an equal volume of water?

What is the specific gravity of air?

NOTE. Keep the bottle and tubes for the next experiment.

Experiment 27. *To find what part of the air contained in the bottle was removed.*

Apparatus. The bottle, stopple, and tubes used in the last experiment; a 250^{cc} graduate.

Directions. Fill the bottle brimful of water. Into the mouth of the bottle, holding the tube upright, put the stopple, and press it into place. Then remove the stopple, and let the water in the tube run into the bottle. (Why?) With the graduate find how many cubic centimeters of water there are in the bottle. In the last experiment you found the number of cubic centimeters of air at atmospheric pressure removed.

What part of the air originally in the bottle was removed?

The quotient just found is called the *degree of exhaustion;* hence the

Definition. *The degree of exhaustion is a number that tells what part of the air contained in a vessel has been removed.* This numerical value is expressed as per cent.

QUESTION. If a vessel of 1000^{cc} capacity, filled with air at atmospheric pressure, has 950^{cc} taken out, what is the degree of exhaustion?
Ans. 95 per cent.

BOYLE'S LAW.

20. Boyle's Law. When we found the specific gravity of air, we made no account of the height of the barometer. The specific gravity of air depends on the pressure of the atmosphere; so it will be interesting to find what relation there is between the pressure of the atmosphere and the volume occupied by a portion of it. The relation is known as Boyle's Law.[1]

[1] In honor of Robert Boyle, who discovered the law in 1662.

Experiment 28. *To find the relation between the volume of a certain mass of air and the pressure put upon this air.*

Apparatus. A wooden support like the one used in Exp. 24; a clean Boyle's tube; clean mercury; a medicine dropper; two rubber bands.

Directions. Fasten the tube to the support by rubber bands (Fig. 18). Get the reading of the barometer. Carefully pour mercury into the tube till the bend is covered and the mercury stands about 3^{cm} higher in the long arm than in the short. Tip the tube to allow some air to escape from the short arm; place the tube upright again; note the levels and, if necessary, repeat the tipping (in the opposite direction this time, perhaps) till the mercury in the closed arm stands 1^{cm} or 2^{cm} above the level of the mercury in the long arm. Then, by means of the medicine dropper, cautiously add mercury till the two levels are the same. In the subsequent parts of the experiment the tube must not be tipped, lest the air in the closed arm be increased or diminished.

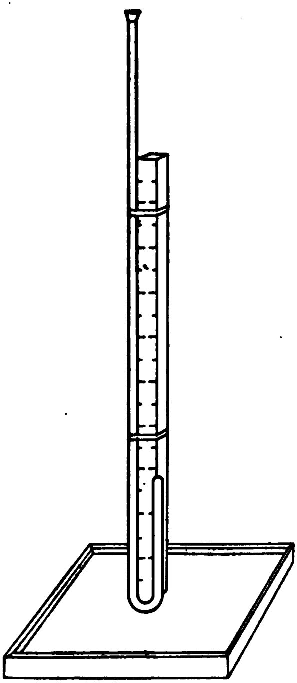


FIG. 18.

Be sure to have the level of the mercury from 1^{cm} to 3^{cm} above the curve of the tube. Read and record the posi-

tion of the top of the meniscus, or rounded part at the end, of each mercury column above the base of the support.

What is now the pressure on the air in the closed arm? [See your inference from Exp. 19, Part 2.]

Now pour in some more mercury, adding the mercury by means of the medicine dropper, globule by globule towards the last, till the level of the mercury in the open arm is half of the barometric height above the level of the mercury in the short arm.

Record the heights of the tops of the columns above the base. In this experiment all measurements of length must be recorded in centimeters.

What is the pressure on the air in the closed branch now?

Pour more mercury into the tube till the level of the mercury in the open arm is just the barometric height above the level of the mercury in the short arm. Record the heights as before.

What is the pressure on the air in the closed arm now?

Finally, if the tube is long enough, make the level of the mercury in the open arm stand one and one-half times the barometric height above the level of the mercury in the closed arm. As before, record heights.

What is the pressure on the air in the closed arm now?

Get the reading of the barometer again. (Why?)

Measure and record the height of the inside of the top of the closed arm above the base of the support.

If you divide each of the last three pressures in your record by the first (the atmospheric) pressure, you can put the quotients into the following form

$$\frac{3}{2}, \frac{4}{2}, \frac{5}{2}.$$

As the tube is of uniform bore, the volume of air in the short arm is proportional in each case to the length of the air column in the short arm, therefore, divide each of the last three lengths by the first and the quotients will be the same as those obtained by dividing the actual volumes. Put these quotients into the fractional form and see how nearly they agree with

$$\frac{2}{3},\ \frac{2}{4},\ \frac{2}{5}.$$

From the results obtained by comparing the quotients of the pressures with the quotients of the corresponding volumes, what do you find to be the relation between the volume of a certain quantity of air and the pressure put upon this air?

What keeps the mercury from filling the short arm?

What principles learned in previous experiments have you made use of in this?

EXAMPLES.

1. A bent tube, having one end open and the other end closed, contains mercury which stands 60^{cm} higher in the open than in the closed branch. Compare the pressure of the air in the closed branch with that of the external air; the barometer at the time standing at 75^{cm}.

Solution. Since the pressure is equally great at all points in a horizontal plane passing through a liquid (see your inference from Exp. 19, Part 2), the pressure at the surface of the mercury in the closed branch (that is, the pressure of the confined air) must equal the pressure at the same level in the open branch; but the pressure at this level in the open branch is the pressure due to the column of mercury 60^{cm} high plus the pressure due to the external air bearing down upon the top of this mercury column; but this air pressure is equal to 75^{cm} (that is, it will support a column of mercury 75^{cm} high); so the total pressure will be

$$60^{cm} + 75^{cm} = 135^{cm}.$$

Thus the pressure of the mass of air confined in the closed branch is 135^{cm}.

$\therefore$ pressure of confined air : pressure of external air $= 135 : 75 = 9 : 5$.

That is, the pressure of the air in the closed branch is $\frac{9}{5}$ as great as that of the external air.

2. A mass of air occupies 100^{cc} when the pressure is 60^{cm}. What volume will it occupy when the pressure is 120^{cm}?

Solution. Boyle's Law states that the volume of a gas varies inversely as the pressure; for example, if a mass of air has a volume V_1 when the pressure is P_1, and a volume V_2 when the pressure is P_2, we shall have the following relation among V_1, V_2, P_1, and P_2:

$$V_1 : V_2 = P_2 : P_1;$$

or, if we call V_1 the first volume, V_2 the second volume, P_1 the first pressure, and P_2 the second pressure, we shall say

1st Volume : 2nd Volume = 2nd Pressure : 1st Pressure.

In the special problem given for solution we have

$$V_1 = 100, \qquad P_1 = 60,$$
$$V_2 = x. \qquad P_2 = 120.$$

So
$$100 : x = 120 : 60,$$
$$120\,x = 6000,$$
$$x = 50.$$

Hence the required volume will be 50^{cc}.

3. A mass of air occupies 200^{cc} when the pressure is 76^{cm}. What must the pressure be in order that this mass of air shall occupy only 25^{cc}?

4. In performing the Torricellian experiment, an inch in length of the tube is occupied with air at atmospheric pressure before the tube is inverted. After the inversion, this air expands till it occupies 15 inches, when a column of mercury 28 inches high is sustained below it. Find the true barometric height.

Solution. In the statement of the problem nothing is said about the area of the cross-section of the tube, so we cannot find the volume of the air that fills an inch in length of the tube; but let us denote the area of this cross-section by a, then a will denote the volume of the air in cubic

inches. After the inversion of the tube, $15a$ will denote the new volume in cubic inches. If x denotes the true barometric height, that is, the pressure of the atmosphere, we know, as was stated in the solution of Example 1, that the pressure at the level of the mercury in the cup, into which the tube was inverted, is equal to the pressure within the tube on the same plane. The pressure on the portion of the plane within the tube is measured by the height of the mercury column above it plus the pressure of the confined air upon the top of this column. The length of this mercury column is given in the statement of the problem as 28 inches. Let us denote by P_2 the pressure of the confined mass of air, then the pressure on the portion of the plane within the tube is $28 + P_2$; but this has just been shown to be equal to the atmospheric pressure x; hence

$$28 + P_2 = x.$$
$$\therefore P_2 = x - 28.$$

Collecting what we know to be true about the volumes and the pressures in this problem, we have

$$V_1 = a, \qquad P_1 = x,$$
$$V_2 = 15a. \qquad P_2 = x - 28.$$

Hence, by Boyle's Law,

$$\begin{aligned} a : 15a &= x - 28 : x, \\ ax &= 15a(x - 28), \\ x &= 15(x - 28), \\ x &= 15x - 420, \\ 14x &= 420, \\ x &= 30. \end{aligned}$$

Hence the true barometric height is 30 in.

NOTE. It will be seen that the a, which we introduced for the purpose of expressing the volume of the air, does not appear in the result. In other words, it does not matter what the area of the cross-section of the tube may be.

5. A tube of uniform cross-section, 200^{cm} long, closed at one end, is pushed open-end downward into a deep cup of mercury till the air within it, which originally filled the whole tube, is reduced to one-half of its first volume. The barometer at the time of performing the experiment reads 76^{cm}. How far below the surface of the mercury in the cup is the surface of the mercury in the tube? How far is the closed end of the tube above the general surface of the mercury in the cup?

6. A quantity of air is collected in a barometer tube, the mercury standing 10 in. higher inside the tube than outside, and the correct barometric height being 30 in. If the volume of the air under these conditions is 1 cu. in., what would be its volume at atmospheric pressure?

Solution. Since the pressure at all points in a horizontal plane passing through a liquid is equally great, we shall have, denoting by P_1 the pressure of the air confined in the tube,

$$P_1 + 10 = 30.$$
$$\therefore P_1 = 20.$$

Collecting what we know to be true about the volumes and the pressures in this problem, we have

$$V_1 = 1, \qquad P_1 = 20,$$
$$V_2 = x. \qquad P_2 = 30.$$

Hence, by Boyle's Law,

$$1 : x = 30 : 20,$$
$$30\,x = 20,$$
$$x = \tfrac{2}{3}.$$

The required volume is, then, $\frac{2}{3}$ cu. in.

7. A quantity of air occupying 10^{cc} is admitted to the space above the mercury column of a barometer, and there expands till it occupies 30^{cc}. The column of mercury beneath this air is now 50^{cm} high. How high was it before the admission of the air?

Suggestion. Denoting by x the required height, show that the pressure of the air in the tube is denoted by $x - 50$. Then collect what you know to be true about the volumes and the pressures, and apply Boyle's Law.

8. The tube of a barometer has a cross-section of $1^{sq\,cm}$, and when the mercury column stands at 77^{cm}, the length of the empty space above it is 8^{cm}; how far will the mercury column be depressed if 1^{cc} of air is passed up into the tube?

Solution. Suppose the mercury to be depressed through x^{cm}; then if we denote by P_2 the pressure of the air confined in the tube, we shall have

$$77 - x + P_2 = 77.$$
$$\therefore P_2 = x;$$

that is, x measures the pressure of this air.

Since the cross-section of the tube is $1^{sq\,cm}$, the volume of this air is $(x + 8)^{cc}$.

Collecting what we know to be true about the volumes and the pressures in this problem, we have

$$V_1 = 1, \qquad P_1 = 77,$$
$$V_2 = x + 8. \qquad P_2 = x.$$

By Boyle's Law, we have

$$\begin{aligned} 1 : x + 8 &= x : 77, \\ (x + 8)\,x &= 77, \\ x^2 + 8x &= 77, \\ x^2 + 8x + 16 &= 77 + 16, \\ (x + 4)^2 &= 93, \\ x + 4 &= 9.65, \\ x &= 5.65. \end{aligned}$$

Hence the mercury column is depressed through 5.65^{cm}.

9. A cylindrical diving-bell, 9 ft. high, is immersed in water so that its top is 27 ft. below the surface. If the height of the water-barometer is 34 ft., find how high the water rises within the bell.

PUMPS.

21. The Air-Pump. We have already used in the laboratory (Exps. 14, 15, and 25) an air-pump of very simple construction. This pump consists (Fig. 19) of a hollow metallic cylinder, *C*, closed at the bottom, where it is supported in a vertical position by an iron foot not shown in the figure. In the cylinder is a piston, *D*, which fits tightly, but which can readily be pushed down and pulled up by a handle attached by the rod, *F*, to the piston. At the lower end of the cylinder are two nozzles,

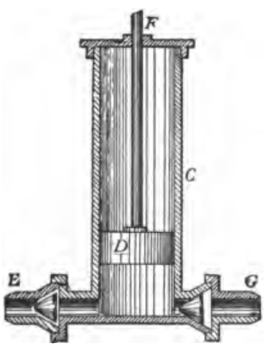


FIG. 19.

E and *G*. When the piston is worked, air is drawn in through one of these nozzles and pushed out through the other. On unscrewing the nozzles, a valve will be found behind each. These valves are made of thin sheet-brass, and are conical in form. They fit closely into conical holes communicating with the interior of the cylinder. The conical holes are called valve seats. In one nozzle the valve seat tapers towards the end of the nozzle, while the valve seat behind the other tapers towards the cylinder.

If the vessel from which the air is to be taken is connected by means of a piece of thick-walled rubber tubing to the nozzle covering the valve whose sharp end points outward, and the piston is raised by pulling up on the handle, the air in the cylinder expands, and thereby its pressure is diminished; so the air in the vessel also expands, and rushes into the cylinder until the pressure becomes equal in both. When the piston is pushed down, the air in the cylinder is compressed. This compressed air closes, by pressing into its seat, the valve by which the air entered the cylinder; but it opens the other valve and escapes into the external air. By repeating the process of raising and lowering the piston, more air is taken from the vessel, till, finally, the difference of pressure between the air in the vessel and the air in the cylinder becomes so small that it is unable to open or close the valve.

No air-pump has ever been made that will give a *perfect* vacuum. In the construction of air-pumps, however, such skill and ingenuity have been used that with a good pump one may obtain a *nearly* perfect vacuum. The bulbs of incandescent lamps which you see in shops and houses

lighted by electricity have the air removed from them by an efficient pump of peculiar construction. In the side and near the top of a long upright tube is an opening to which the lamp-bulb is attached at a certain stage in its process of manufacture. Mercury is then poured down the tube. As the mercury passes the opening, the air in the bulb expands and is swept down the tube.

22. The Common Lifting-Pump. This pump, so common in houses and yards, is used for raising water from a well or cistern to the surface of the ground. The construction of this pump is as follows:

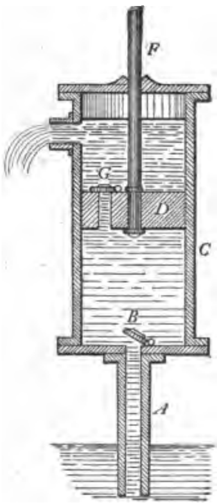


FIG. 20.

A long pipe, *A* (Fig. 20), extends from the surface of the ground to some distance below the surface of the water in the well. At its upper end the pipe has a valve, *B*, hinged like a trap-door, opening upwards. To the upper end of the pipe is fastened a hollow cylinder, *C*, in which is a tightly fitting piston, *D*, that can be raised and lowered by lowering and raising the pump-handle to which it is attached by means of a rod, *F*. In the piston is a valve, *G*, opening upwards like that in the pipe. We shall explain the action of the pump by supposing the part of the pipe above the water in the well to be filled with air at the start. On pushing down the piston by raising the pump-handle, the air in the cylinder is compressed. This com-

pressed air pushes down on the valve, *B*, and closes it more tightly; but by pushing up on the valve, *G*, in the piston, the compressed air opens it, and continues to escape as long as the piston descends. When the piston is raised, the air in the cylinder becomes rarefied; the atmosphere presses down on the valve in the piston and closes it tightly, but the air in the tube pushes open the valve, *B*, and enters the cylinder. By again lowering the piston and raising it, some more air is taken out. In the meantime the pressure of the atmosphere on the water in the well forces the water up the tube, till, as the action of the pump is continued, the water enters the cylinder. Now, when the piston is pushed down, the water rises through the piston-valve, *G*; when the piston is raised again, the piston-valve closes, and the valve in the pipe is pushed open by the water forced into the cylinder by the pressure of the atmosphere. The piston lifts the water above it until the spout is reached, whence the water flows into the pail placed to receive it. Each succeeding stroke brings a fresh supply of water, which keeps a nearly continuous stream flowing from the spout.

It frequently happens that the valves do not fit accurately, and in such cases it is necessary, in starting the pump, to pour some water into the cylinder above the piston.

Question. When the mercury barometer column stands at a height of 30 inches (specific gravity of mercury = 13.6), what is the greatest height to which water can be raised by the common lifting-pump?

23. The Force-Pump. This is the pump employed when water is to be raised to a height above the pump, or when a forcible stream is desired. In construction this

pump resembles the lifting-pump, but the piston, instead of having a valve in it, is solid; and a valve, *G*, is placed at the mouth of a pipe that leads from a point near the bottom of the cylinder to the place where the water is to be delivered. The valve, *B*, at the top of the pipe is placed in the same position as the corresponding one in the lifting-pump. The valve in the pipe leading from the side of the cylinder opens away from the cylinder (Fig. 21); consequently, when the piston is raised, this valve closes (Why?), and the air below the valve, *B*, in the pipe leading into the well forces its way through and rushes into the cylinder to equalize the pressure. By continuing the process of pumping, the air is at last removed from the pipe, and the water in the well is forced, by the atmospheric pressure on the surface of the water in the well, into the cylinder; then the piston, when lowered, pushes down on this water and forces it through the valve, *G*, into the side-pipe; when the piston is again lifted, more water is raised from the well into the cylinder. (Why does not the water that has just been forced into the side-pipe run back into the cylinder?) Then, lowered again, the piston forces the water in the cylinder into the side-pipe, which conveys the water, as the pumping goes on, to the place where it is to be delivered.

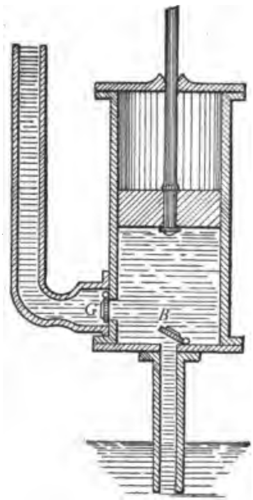


Fig. 21.

In the steam fire-engine and other force-pumps the side-tube leads into a reservoir of air; the air becomes somewhat compressed by the water and sends it out in a continuous stream.

THE SIPHON.

24. The Siphon. The siphon (Fig. 22) in its simplest form is a bent tube open at both ends. It is used for transferring a liquid from one vessel to another when it is not convenient to make a hole in the side of the vessel or to tilt it so that the liquid shall run out. To make the siphon begin working, one end is dipped into the liquid which we wish to remove, and suction, by means of the mouth or otherwise, is applied to the other end. When the liquid is corrosive, like a strong acid, the method of procedure should be to fill the siphon with some of the liquid, close one end, and dip the other into the vessel of liquid; on uncovering the end, the liquid will at once begin to flow. In order that the siphon may work, it is necessary, if the liquid is discharged into the air, that the end, *A*, from which the liquid flows should be below the level, *EF*, of the liquid in the vessel, or, if the liquid is discharged into another vessel, the surface of the liquid in this vessel must be at a lower level than that in the former.

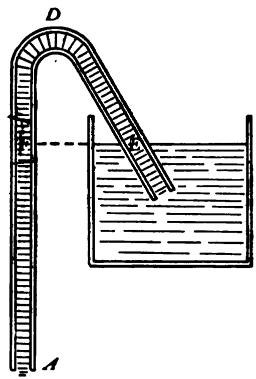


FIG. 22.

The explanation of the action of the siphon, the reason why the liquid flows first up hill, from *E* to *D*, and then down hill, from *D* to *A*, is as follows:

The pressure inside the arm, *DA*, of the siphon, which does not dip into the vessel containing the liquid to be removed, at a point, *F*, lying in the same plane as the surface of the liquid, is the same as at the point, *E*, correspondingly situated in the other arm; but this pressure is the same as the atmospheric pressure at a point on the surface of the liquid in the vessel (Why? See your inference from Exp. 19, Part 2). The pressure of the liquid at the extremity, *A*, of the arm which is outside the vessel is greater than at the point, *F*, already spoken of, lying on a level with the surface of the liquid (Why? See your inference from Exp. 17); but the pressure at this point has already been shown to be equal to the atmospheric pressure; hence the pressure at the extremity, *A*, of the tube is greater than the atmospheric pressure, and, consequently, the liquid pressure at this point overcomes the atmospheric pressure from without, and the liquid tends to separate at the highest point, *D*, of the siphon and to run out. If the height of the bend, *D*, of the siphon above the level of the liquid in the vessel is less than the height at which the liquid would stand in a barometer tube, the pressure of the air will prevent the liquid from separating at the bend, *D*, and by forcing the column of liquid to ascend in the arm dipping into the liquid, will maintain a continuous flow.

How long will the liquid continue to flow after the level of the liquid in the vessel into which the siphon discharges becomes the same as that of the liquid in the first vessel?

QUESTIONS. When the barometer stands at a height of 76^{cm}, what is the greatest height over which mercury can be carried by means of a siphon? When the barometer stands at a height of 30 inches (specific gravity of mercury = 13.6), show by computation that the greatest height over which a siphon can raise water will not exceed 34 feet.

THE HYDROSTATIC PRESS.

25. The Hydrostatic Press. The hydrostatic, or, as it is sometimes called, the hydraulic press, is a machine

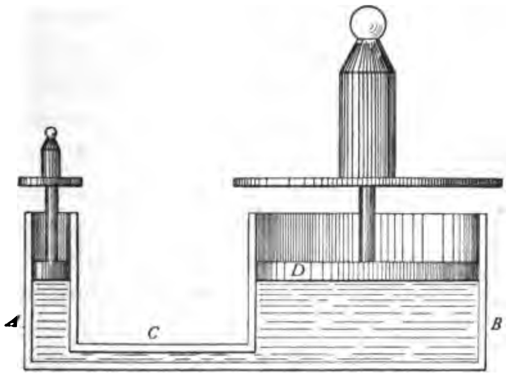


FIG. 23.

for lifting heavy weights, and for compressing merchandise into small compass. This machine is in its essential principles constructed as follows:

Two hollow cylinders, *A* and *B* (Fig. 23), of iron, each closed at one end, are set side by side in a vertical position with their open ends uppermost; a pipe, *C*, of iron connects the two near the bottom, so that water poured into one cylinder will flow through this pipe into the other. These cylinders are of unequal area of cross-section, the

larger being, perhaps, 1000 times that of the smaller. From Exp. 23, you already know that if water is poured into one of the cylinders this water will rise to an equal height in both. If a pressure be applied by means of a tightly fitting piston to the top of the water column in the smaller cylinder, it will take a much greater pressure applied to the top of the water column in the larger to balance this pressure; thus, if a pressure of 1 lb. were applied to the piston of the smaller cylinder, a pressure of 1000 lbs. would have to be laid on the piston, *D*, that rests on the top of the water column in the larger cylinder. The explanation of this fact is that water transmits a pressure applied to it with undiminished force in all directions; so that when the pressure of 1 lb. is applied to the top of the smaller water column, the water transmits this pressure to each portion of the inner surface of the two cylinders; consequently each portion of an area equal to that of the smaller piston has now upon it an additional pressure of 1 lb. As the area of the face of the larger piston is 1000 times that of the smaller, the pressure on this face will be 1000 lbs. The larger piston is then forced upwards with a pressure of 1000 lbs. Hence, to keep the larger piston from moving up, it would have to be loaded with a weight of 1000 lbs.

A strong framework is built above the larger cylinder, and the substance that is to be compressed is put between this and the piston. Had the area of the cross-section of the larger cylinder been 10,000 times the area of the cross-section of the smaller, a pressure of 1 lb. applied to the smaller piston would produce a pressure of 10,000 lbs. on the larger piston.

The hydraulic press is widely used in the arts. It may be constructed to give pressures of two or three hundred tons.

MARIOTTE'S BOTTLE.

26. Mariotte's Bottle. This piece of apparatus (Fig. 24) consists of a bottle having three small holes drilled through its sides, one near the top, one near the bottom, and the third half-way between the other two. A rubber stopple with one hole has a glass tube thrust through it; this tube is long enough to reach to the bottom of the bottle when the stopple is put in place, and also to project 10^{cm} above the top of the stopple. This piece of apparatus is very useful in fixing the student's ideas about liquid pressure and its consequences; so the following experiment should be carefully performed.

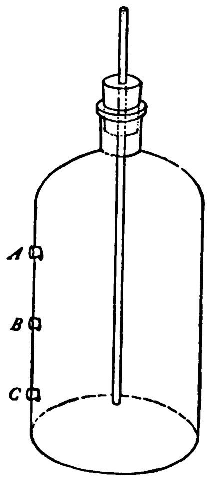


FIG. 24.

Experiment 29. *To find what will happen on removing one or more stopples in the side of a Mariotte's bottle filled with water.*

Apparatus. A Mariotte's bottle; water.

Directions. After securely stopping the holes in its sides by means of small cork stopples, as shown in Fig. 24, fill the bottle brimful of water; then push the tube and stopple into place so that the water will rise in the tube and stand at some distance above the stopple. Be careful not to have any air-bubbles in the bottle.

Before removing any of the side stopples, as you will be directed to do, call to mind the facts you have already learned about liquid pressure in Exps. 17, 18, and 19, and try to predict what will take place; then, having made up your mind as to what will take place, remove the stopple, and see if your prediction is verified.

After placing a vessel to catch the water, take out the stopple that closes the hole, *A*, highest up in the side of the bottle; then replace this stopple and remove the one at *B*; finally replace the stopple at *B* and remove the one at *C*.

Does the water run out freely in every case?

At what level does the water in the tube stand after the removal of each stopple?

Now, without removing the large stopple, slip the tube up till its lower end is on a level with *A*; open *A* and allow all the water that will run out to do so. Refill the bottle, push the tube down till its lower end is on a level half way between *A* and *B*, and remove the stopple from *A*; then close *A* and let the water run from *B* a short time. Push the tube down till its lower end is on a level between *B* and *C*; have the bottle completely filled with water and remove the stopple at *A*. Replace the stopple at *A* and remove the one at *B*. Replace the stopple at *B* and remove the one at *C*.

In order to have water flow freely from the bottle, what relation must there be between the position of the hole from which the flow takes place and the end of the tube?

Fill the bottle once more, and keeping the end of the tube below *C* remove the stopple at *C*, and then take out also the stopple at *A*.

When both stopples are out, what takes place at *C*?

When both stopples are out, what takes place at *A*?

Write out a brief explanation of the facts you have observed.

GRAPHICAL METHOD.

27. Graphical Method of Representing Results. In Exp. 28 we obtained a series of volumes of air, which decreased in proportion as we increased the pressure applied to the confined mass of air. Whenever, as in this instance, we have a series of results which depends on a uniform variation of some one circumstance in the experiment, the relation among the results can be clearly represented to the eye by employing what is known as the *graphical method*. To illustrate the application of this method, let the following be the data obtained, as in Exp. 28:

Pressure.	Volume.
1	100^{cc}
1.5	66.6
2	50
2.5	40
3	33.3
3.5	28.6
4	25

The first column gives in atmospheres the pressures, which have been increased by the addition of half an atmosphere at a time; the second column gives the volume corresponding to each pressure.

On a sheet of coördinate paper, which is ruled into little squares by equally spaced lines, a horizontal line is chosen

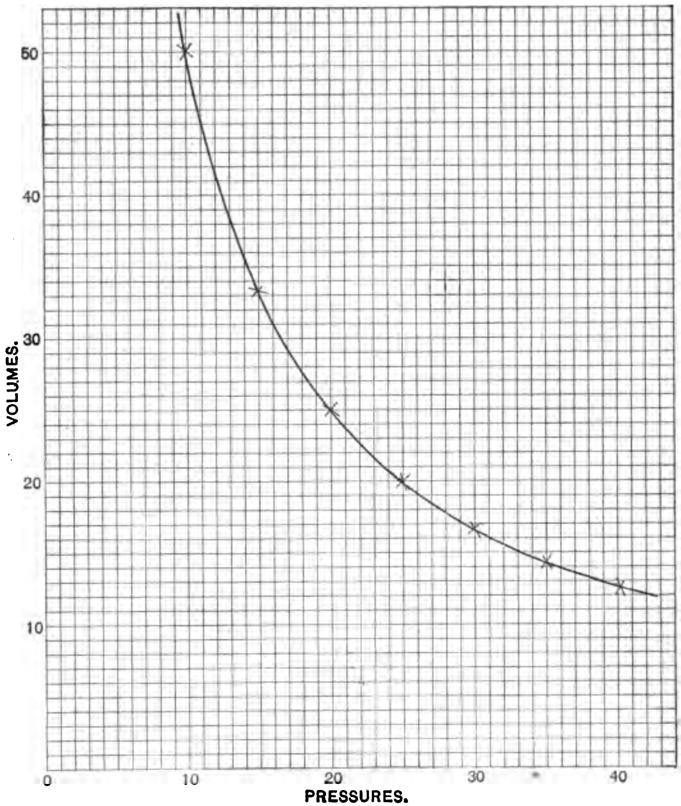


FIG. 25.

as the "axis of abscissae," and a vertical line as the "axis of ordinates"; the point at which these two lines intersect is called the "origin." Starting from the origin, there is

laid off on the horizontal line a certain distance, equal, say, to 10 of the divisions, as in Fig. 25, to represent the first pressure, and from the point thus reached a vertical distance is measured, equal, say, to 50 of the divisions, to represent the corresponding volume; the end of this vertical line is marked by a point. Each of the remaining observations is treated in the same way as the first, by using the same scale as was at first employed for measuring distances in each of the directions (that is, if 10 divisions represent 1 atmosphere, 15 divisions represent 1.5 atmospheres; if 50 divisions represent 100^{cc}, 33.3 divisions represent 66.6^{cc}). Finally, a strip of hard rubber or a steel spring is laid edgewise upon the paper, and made to pass through the points marking the extremities of the vertical lines. A line is drawn through these points with a sharp lead-pencil.

This line represents to the eye the relation existing among the various observations. When the numerical data employed in the construction of this line have been carefully determined, and the line itself drawn with precision, it is possible to obtain with ease and accuracy the volume the air will occupy for any given pressure between the extreme pressures made use of in this experiment. To do this, count from the origin on the horizontal line a number of divisions corresponding to the given pressure; then count the number of divisions in the vertical line lying between this point and the line joining the extremities of the vertical lines. The number thus obtained will represent the required volume on the scale employed in representing the volumes; in the present case two divisions on a vertical line represent unit volume.

Let the student construct on a piece of coördinate paper a curve in the manner just described, using the results obtained in Exp. 28.

EXPERIENCE.

28. Experience; Observation; Experiment. At this stage of our progress it may be well for us to think a little about the first thing necessary for the successful study of physics. This necessary thing is *experience*, which furnishes us with facts. We obtain *experience* by either *observation* or *experiment.*

In observation we merely note and record phenomena (that is, things which appear). Thus, a U. S. Signal Service man observes the ever-changing weather, and notes the height of the barometer, the temperature and moisture of the air, the direction and force of the wind, the height and character of the clouds.

In *experiment* we vary at will the conditions under which phenomena occur. We might have to wait years, or even centuries, to meet with facts which we can readily produce at any moment in a laboratory.

The work of the U. S. Signal Service man is confined to pure observation, for he is unable to control any of the phenomena that he studies. The work of the physicist and the chemist is not confined to observation alone; it also includes experiment, for each is able to vary at will the conditions under which many of the phenomena that he studies occur. Where it can be employed, *experiment* is a fruitful and direct means of getting facts.

Experiments are of two classes: *qualitative* and *quantitative.*

A *qualitative* experiment has for its object the *production* and *observation* of phenomena.

A *quantitative* experiment has for its object the *measurement* of the *magnitudes* of the phenomena produced.

Exp. 19 is a qualitative experiment; Exp. 21 is a quantitative experiment. Divide the 29 experiments you have performed into qualitative and quantitative experiments, and make a list of each group.

29. Facts and Inferences from Facts. On the left-hand page of his note-book the beginner must constantly try to record only what his senses have actually observed. He is apt to record his own character and feelings rather than the facts, for the mind is like an uneven mirror, and does not reflect the facts without distortion. Since the mind by long experience has acquired the power of judging unconsciously of many things of which his senses have not actually assured him, the beginner confuses facts observed with inferences from these facts.

EXAMPLES.

1. Find the specific gravity of a body that weighs 58^g in air and 40^g in water.

2. A specific gravity bottle holds 100^g of water and 180^g of sulphuric acid. Find the specific gravity of the acid.

3. Find the volume of a solid that weighs 357^g in air and 253^g in water.

4. A glass ball loses 33^g when weighed in water, and loses 6^g more when weighed in a saline solution. Find the specific gravity of the solution.

5. A body lighter than water weighs 102^g in air; and when it is immersed in water by the aid of a sinker, the joint weight is 23^g. The sinker alone weighs 50^g in water. Find the specific gravity of the body.

6. Find the specific gravity of kerosene from the following data: In a U-tube the length of the kerosene column is 100^{cm}, and the length of the water column just balancing this is 79^{cm}.

7. Find the volume of a solid which weighs 458^g in air, and 409^g in brine. Specific gravity of the brine = 1.2.

8. How much weight will a body lose whose volume is 47^{cc}, when weighed in a liquid the specific gravity of which is 2.5?

9. How much water will 1000^{cc} of oak displace when floating in equilibrium? Specific gravity of oak = 0.8.

10. Why is it, in the metric system of weights and measures, that the specific gravity of a substance and the numerical value of its density are the same?

11. Define *density;* define *specific gravity.* A certain solid floats in water with only two-thirds of its volume submerged. What is the specific gravity of this solid?

12. A pump is used to draw water from a well through a vertical pipe. How long may the pipe be, the barometer reading 76^{cm}, and the specific gravity of mercury being 13.6? Tell and explain what would happen if a small hole were bored in the side of the pipe, when full of water, at a point half-way up.

13. When the height of the column of mercury (sp. gr. = 13.6) in the barometer is 80^{cm}, how tall a column of water can the pressure of the atmosphere support in a tube having a perfect vacuum at the top?

14. A rectangular block 10^{cm} thick floats in water with 6^{cm} of its depth submerged. Find the specific gravity of the block, showing your reasoning.

15. From the following data find the specific gravity of sulphuric acid:

Weight of bottle empty	$= 50^g$.
Weight of bottle filled with water	$= 150^g$.
Weight of bottle filled with sulphuric acid	$= 234^g$.

16. How much would the result obtained in the last example have been changed if 1^{cc} of the bottle had been left empty when it was weighed with water?

17. A cork of sp. gr. 0.25, the volume of which is 10^{cc}, floats upon mercury of sp. gr. 13.6. How great is the submerged part of the cork?

18. Given that a body appears by the indications of a spring balance to weigh 15^g in air and 10^g in water. Answer the following questions:

(*a*) If the balance is correct, what is the specific gravity of the body?

(*b*) If each reading of the balance is only $\frac{9}{10}$ as large as it should be, what is the specific gravity of the body?

19. A cubical vessel, each side of which is 10^{cm} square, has a tube $1^{sq\,cm}$ in cross-section and 20^{cm} tall rising from the middle of its top. The tube is open at both ends, and both vessel and tube are full of water. Neglecting atmospheric pressure and weight of vessel and tube, find

(*a*) The total pressure which the vessel and tube as now filled exerts upon the support.

(*b*) The total pressure exerted against the bottom of the vessel by the water within it and the tube.

If the pressures are not equal, explain the difference.

20. A vessel is filled with water to a depth of 40^{cm}. A cylinder of wood 30^{cm} long and $100^{sq\,cm}$ in area of cross-section, the specific gravity of which is 0.5, extends upward through a hole in the bottom of the vessel, the top of the cylinder being 20^{cm} beneath the surface of the water. Show whether the cylinder tends to rise or fall, and how great a force is required to hold it in its present position.

21. A block, the density of which is to be determined, is measured with a scale the divisions of which are only $\frac{9}{10}$ as long as they are supposed to be. How much too small or too large will the value found for the density be in consequence of this error?

Solution. Let a, b, and c denote the true dimensions of the block, and let W denote its weight. Then the true density, d, will be

$$d = \frac{W}{abc}.$$

As each of the divisions of the faulty scale with which the block is measured is $\frac{9}{10}$ of what it is supposed to be, that dimension of the block denoted by a will, according to this scale, have a value equal to the number of times a contains $\frac{9}{10}$, that is,

$$\frac{a}{\frac{9}{10}}, \text{ or } \frac{10}{9}a.$$

In like manner, the dimension denoted by b will have a value $\frac{10}{9}b$ when measured by the scale, and the dimension denoted by c will have a value $\frac{10}{9}c$. The product of $\frac{10}{9}a$, $\frac{10}{9}b$, and $\frac{10}{9}c$, that is, $\frac{1000}{729}abc$, will

denote the false volume found by using the measurements indicated by the faulty scale; and d', the false density, will be

$$d' = \frac{W}{\frac{1000}{729} abc} = \frac{729}{1000} \frac{W}{abc}.$$

The difference between the true density and the false is

$$d - d' = \frac{W}{abc} - \frac{729}{1000} \frac{W}{abc} = \frac{1000}{1000} \frac{W}{abc} - \frac{729}{1000} \frac{W}{abc} = \frac{271}{1000} \frac{W}{abc};$$

or, in words, the value found for the density will be too small by 0.271 of the true density.

22. A block, the density of which is to be determined, is measured with a scale the divisions of which are $\frac{4}{5}$ as long as they are supposed to be. How much too small or too large will the value found for the density be in consequence of this error?

23. A cube of wood 10^{cm} on each edge, and of sp. gr. 0.5, is covered on one side by a piece of metal 10^{cm} square and 1^{cm} thick, of sp. gr. 5. How deep would the whole mass of wood and metal sink in water?

Solution. Volume of wood $= 1000^{cc}$.
Volume of metal $= 100^{cc}$.
Volume of metal and wood $= 1100^{cc}$.

Weight of wood $= 0.5 \times 1000 = 500^{g}$.
Weight of metal $= 5 \times 100 = 500^{g}$.
Weight of metal and wood $= 1000^{g}$.

Since the volume of the wood and the metal together is 1100^{cc}, and their weight only 1000^{g}, the composite block will float with 1000^{cc} of its volume beneath the water. (Why?)

The composite block would naturally float with the metallic plate downward; hence, the outer face of the metallic plate would be 10^{cm} below the surface of the water.

24. A cylinder 20^{cm} long consists of a cylinder of iron, sp. gr. 7, 1^{cm} long, and one of wood, sp. gr. 0.5, 19^{cm} long. If this cylinder floats upright in water, how many centimeters of its length will be above the water?

25. A cube of wood, 10^{cm} on an edge, floats in water with 8.5^{cm} of its depth submerged. How many cubic centimeters of its volume will be under water after kerosene, of sp. gr. 0.8, is poured into the vessel containing the water till the block is completely submerged?

26. A piece of wood, of volume 1200^{cc}, floats with two-thirds of its volume beneath the surface of the water. What is the least number of cubic centimeters of lead, of sp. gr. 11.3, that must be attached to the block to submerge it completely?

27. A long tube closed at one end is partly filled with mercury, and the remainder with air. When this tube is inverted in a deep cup of mercury and pushed down till there is only 50^{cm} of its length projecting above the surface of the mercury in the cup, the level of the mercury in the tube and in the cup is the same. When the tube is raised till 100^{cm} of its length projects above the level of the mercury in the cup, the surface of the mercury in the tube stands 25^{cm} above the level of the mercury in the cup. What is the pressure of the atmosphere?

28. A barometer which has air in the space above the column reads 65^{cm}. If the distance from the level of the mercury in the cistern to the top of the barometer tube is 85^{cm}, and if the air in the tube has a volume of 1^{cc} when the pressure is 77^{cm}, find the true barometric height, the area of the cross-section of the tube being $1^{sq\,cm}$.

CHAPTER II.

HEAT.

30. Some Effects of Heat. We shall begin the study of heat with some experiments for the purpose of finding out whether heat will change the size of bodies.

Experiment 30. *To find whether heat will change the size of a volume of water.*

Apparatus. A bottle having a wide mouth, like the one used in Exp. 11; a good cork stopple to fit the bottle; a piece of glass tube 30^{cm} or 40^{cm} long, and 0.3^{cm} or 0.4^{cm} in diameter; a Bunsen burner; a copper boiler; two little rubber bands; a jar of cold water.

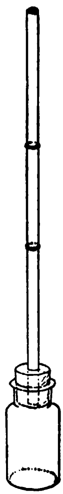


FIG. 26.

Directions. Under the boiler half full of water place the lighted Bunsen burner. While the water is getting warm, pierce a hole through the stopple with a cork borer a little smaller than the tube. Push the tube through the cork. Fill the bottle brimful of water, and, taking care not to enclose air-bubbles, press the stopple into the mouth of the bottle. If the water fills the tube completely, shake out some, so that the water will fill less than half the tube. Around the tube (Fig. 26), at the top of the water column, slip a rubber band for a marker. Have the water in the boiler so hot as to be uncomfortable to the finger held in it for an instant; if too hot, add cold water. Put the bottle into the hot water, and for two or three minutes watch the

column of water in the tube. At the end of the two or three minutes, put the other rubber band on the tube, at the top of the water column.

What have you observed while you have watched?

What is meant by expansion?

When heated, does the water in the bottle expand?

Put the bottle into the jar of cold water, and for three or four minutes watch the water column.

What do you observe?

What is meant by contraction?

When cooled, does the water in the bottle contract?

NOTE. This bottle of water with the tube might be used to tell us which of two liquids is the warmer; for we might put the bottle first into one liquid and then into the other, and in each case observe the height of the water column in the tube. The water column would stand higher for the warmer liquid. The comparison could be more accurately made by means of a peculiar instrument, the thermometer (from two Greek words *therme*, warmth; and *metron*, a measure), which in action, and somewhat remotely in form, resembles our bottle and tube.

Experiment 31. *To find whether water, alcohol, and ether, when equally heated, expand equally.*

Apparatus. Three bulb-tubes, with two little narrow rubber bands on each stem as markers; water, alcohol, ether; a copper boiler.

NOTE. The teacher should fill one bulb and a part of its stem with water, one with alcohol and one with ether, and should clamp the bulb-tubes in a group, as shown in Fig. 27, to a retort-stand which is firmly fastened to a table. There should be no flame near this apparatus, as the vapor of ether is inflammable, and, with air, forms an explosive mixture.

Directions. Have the copper boiler three-quarters full of water a little warmer than the air of the room. On

each tube, which should have a label as in Fig. 27, adjust a marker to a level with the surface of the liquid within. Place the boiler so that the bulbs shall be beneath the surface of the water. *Move neither the retort-stand nor the bulbs.* Watch the columns for two or three minutes.

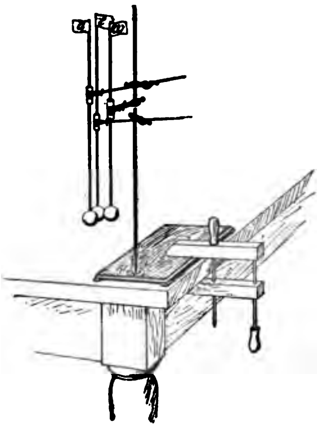


FIG. 27.

Make a table, by putting down the names of the three liquids, showing the relative expansion.

Experiment 32. *To find whether heat will make air expand.*

Apparatus. The bottle, stopple, and tube of Exp. 30; a jar of water.

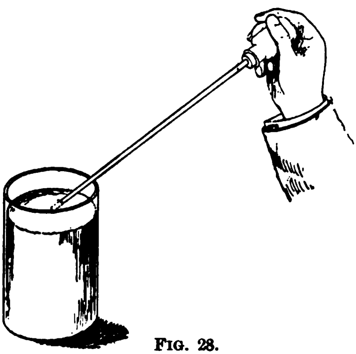


FIG. 28.

Directions. Put the stopple, through which the tube has been thrust, tightly into the mouth of the bottle, which should contain nothing but air. Clasp the bottle in the hand, covering it more completely than is shown in Fig. 28, and push the end of the tube about 1^{cm} below the surface of the water in the jar. For one or two minutes watch the water at the end of the tube.

What happens in the water at the end of the tube?

How do you account for what takes place?

Without removing the end of the tube from the water, take the hand from the bottle. For one or two minutes watch the water in the end of the tube.

What does the water in the end of the tube do?

How do you explain this phenomenon?

MERCURY THERMOMETER.

31. The Thermometer. Upon the table before you place a mercury thermometer, and make an accurate sketch of it in your note-book.

Observe that the thermometer consists of three principal parts:

1. A spherical or elongated vessel containing mercury. This vessel, called the *bulb*, corresponds to the bottle of Exp. 30.

2. A glass tube of small bore partly filled with mercury. This tube, called the *stem*, corresponds to the tube joined to the bottle of Exp. 30. The thread of mercury partly filling the stem is called the *column*. The position of the top of the column gives the *height* of the thermometer.

3. A *scale* made by scratches on the stem, or made of paper and enclosed in a glass tube which also contains the stem. This scale is divided into equal parts, and each one of these parts is called a *degree of temperature*.

You will probably notice in some thermometers a little bulb or slight enlargement at the top of the stem. This little bulb, if the thermometer is overheated, catches the mercury, which would otherwise press against the end of the tube and break it. Before using a thermometer, be

sure to examine this place, for, when the thermometer happens to get turned upside down, a little mercury frequently lodges here. If there should happen to be any mercury in the little bulb, grasp the stem at the end, where the little bulb is, firmly in the hand. Extending the arm to its full length, raise the hand so that the large bulb shall point towards the ceiling. Then, carrying the hand forward, with a quick sweep, bring it to its natural position by the side, thus making the large bulb trace out a semicircle in the air. A single energetic treatment is usually sufficient to dislodge the mercury.

Before using a thermometer, look carefully also along the stem at the column, which is sometimes broken, that is, separated into two or more parts. In case the column is broken, firmly grasp the stem, with the bulb downwards, at its middle part in the fingers of the right hand. Keeping the stem vertical, raise the hand a little way, and then bring the hand down sharply upon the palm of the left hand. In most cases this treatment once or twice repeated suffices to mend the column.

On that part of the scale lying to the right-hand side of the stem you will notice in many thermometers a column of ciphers; while on the left-hand side you will see a column of figures. A cipher on the right-hand side belongs with the figure opposite, so we have 10, 20, 30, etc. The space between 10 and 20, 20 and 30, etc., is divided into 10 equal parts, or degrees.

The thermometer that we have described is called a chemical thermometer with a Centigrade scale. This is the thermometer which we shall use in all our work in heat.

Experiment 33. *To find what temperature a thermometer indicates when placed in melting ice.*

Apparatus. A thermometer; a beaker full of snow or broken ice.

Directions. In a vertical position, in the beaker of broken ice, put the thermometer so that its bulb and a part of its stem shall be covered. Watch the column. After a time the column ceases to fall and comes to rest. The column is now said to be *stationary*. By repeatedly trying this experiment, physicists have found that the temperature of clean ice, when *melting*, is always the same.

Why does the column fall?

From the position of the column, what symbol should you infer is used to indicate the temperature of melting ice?

Experiment 34. *To find the temperature a thermometer indicates when placed in boiling water.*

Apparatus. A thermometer; a beaker; a retort-stand and ring; a piece of wire gauze about 15^{cm} square; a Bunsen burner.

Directions. After wiping the beaker dry on the outside, set it half full of water on the wire gauze laid on the ring of the retort-stand. Place the lighted Bunsen burner beneath to heat the water. Dip the thermometer bulb into the water, and watch the column until the water boils.

Does the column become stationary soon after the water begins to boil?

NOTE. Physicists have found that the temperature of boiling water, or, more accurately, the temperature of the *steam* from boiling water, is constant only when certain conditions are fulfilled. These conditions we shall soon consider.

32. The Fixed Points. By Exps. 33 and 34 you have become acquainted with two remarkable points of the thermometer scale, the *melting-point* of ice, commonly called the *freezing-point;* and the *boiling-point* of water. These two points are called *fixed points*, as each of them, under proper conditions, represents an invariable temperature.

On the Centigrade scale, the space between the fixed points is divided into 100 degrees. Divisions are often carried along the scale below the freezing-point and above the boiling-point. Divisions below the point marked zero (0) are indicated by the negative sign; thus, —10° C. means 10 degrees below the freezing-point (0) on the Centigrade scale. The mark ° stands for degree or degrees.

33. Heat and Temperature. The terms *heat* and *temperature* are used in physics so frequently and are of such importance that particular attention will be given to the meaning of these terms in the following experiment.

Experiment 35. *To find whether heat and temperature are the same.*

Apparatus. A Bunsen burner; a retort-stand and ring; a piece of wire gauze about 15^{cm} square; two beakers of equal size; a thermometer.

Directions. Fill one of the beakers one-third full of cold water, the other two-thirds full. Place them side by side on the gauze laid on the ring of the retort-stand. Put the Bunsen burner beneath, in a position to heat both beakers equally.

When the water begins to boil in one of the beakers, find its temperature with the thermometer, and immedi-

ately afterward get the temperature of the water in the other beaker.

Is the temperature higher in one beaker than in the other?

If the same quantity of heat had entered each beaker, what inference can you draw?

Does a thermometer indicate the quantity of heat a body contains?

Definition. *Heat is that which is capable of producing in us the sensation of warmth.*

Definition. *The temperature of a body tells us the intensity of the heat in it.*

TEMPERATURE AND PRESSURE.

34. The Relation between the Temperature at which Water boils and the Amount of Atmospheric Pressure. On high mountains, like those in Colorado, one must boil an egg for five minutes in order to cook it as hard as if it had been boiled for three minutes at the seashore. If we really knew that water when boiling on a mountain has a lower temperature than when boiling at the seashore, this fact could be easily explained.

As it is not convenient for us to climb a mountain, boil water on the mountain side, and test the temperature of this boiling water with a thermometer, we shall bring about in the laboratory conditions similar to those at the top of a mountain. The pressure of the air on a mountain is less than the pressure at the foot of the mountain, so in the next experiment we shall get the temperature of water when it is boiling under a reduced pressure.

Experiment 36. *To find what influence a diminution of pressure has upon the boiling-point of water.*

Apparatus. A 250^{cc} Kjeldahl flask; a rubber stopple with two holes to fit the flask; a thermometer; a piece of glass tube about 15^{cm} long, bent at right angles, and of a size to fit one of the holes in the stopple; a piece of pressure tube about 10^{cm} long; a Bunsen burner; an air-pump; a piece of pressure tube about 40^{cm} long, with a pointed glass tube in one end; a retort-stand with two rings.

Directions. Place the retort-stand beside the air-pump, and over the exhaust nozzle slip the end of the pressure tube. Have the flask half full of water. Through one hole of the stopple push the bent tube; through the other, the thermometer, so that its bulb will be 2^{cm} or 3^{cm} from the bottom of the flask when the stopple is in place. Insert the stopple, and over the end of the glass tube slip the rubber tube. See that there is a free exit through the tube from the inside of the flask to the open air, lest there should be a *dangerous explosion* on heating the flask.

Hold the flask by the extremity of its neck, and, avoiding an exposure to the flame of any part of the vessel not covered by water, heat gradually. When the water boils, note the temperature indicated by the thermometer. In case the neck becomes clouded with mist, making it impossible to read the thermometer, throw a dry cloth or towel around the flask, and, by tipping it, cause a little water to run into the neck to wash away the mist. Take the flask to the air-pump, and support it, as shown in Fig. 29, by the rings of the retort-stand. By inserting into the piece of pressure tube on the flask the pointed bit of glass tube attached by the long rubber tube to the air-pump, connect the flask and the air-pump.

Before you begin to pump, record the temperature of the water.

Is the water boiling, that is, are bubbles rising through the mass of the water and breaking at the surface?

Make one or two strokes of the pump.

What happens in the flask?

After two or three minutes make one or two strokes more of the air-pump.

What happens in the flask?

Record the temperature the water now has.

From the facts obtained in this experiment, what inference can you draw?

Why does it take a longer time to cook an egg on a mountain than at the sea-level?

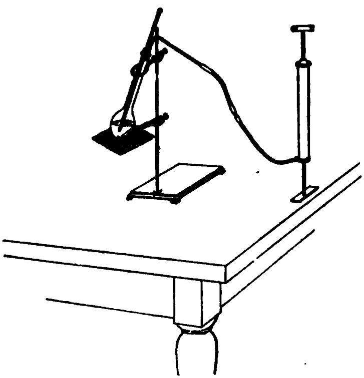


FIG. 29.

Experiment 37. *To find what influence an increase of pressure has upon the boiling-point of water.*

Apparatus. With the exception of the air-pump, the same as in the last experiment; also, a retort-stand with two rings; a piece of wire gauze 15^{cm} square; a glass tube bent at right angles, with one arm about 30^{cm} long and the other about 5^{cm}; an iron pan; a test-tube; mercury; a cloth; blotting-paper.

Directions. Put the flask half full of water upon the wire gauze laid on one of the rings of the retort-stand. Over the neck of the flask pass the other ring, and clamp it to the retort-stand to keep the apparatus from tipping

over. Slip the short arm of the tube into the bit of pressure tube attached to the stopple. Put the stopple into the flask, and heat the water to boiling. Record the temperature of the boiling water. If necessary, wash away the mist as in the last experiment.

Pour mercury into the test-tube to the depth of about 3^{cm}. When the steam is coming freely from the end of the glass tube, wrap a cloth round the test-tube, and, holding it over the pan, not shown in Fig. 30, push the end of the glass tube to the bottom of the test-tube.

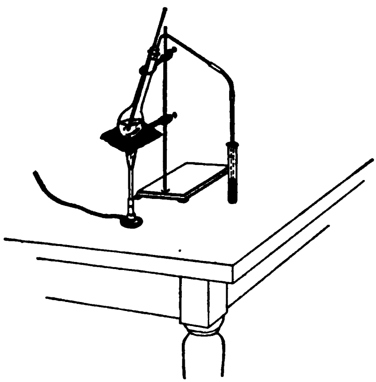


FIG. 30.

Have you increased the pressure in the flask?

Watch the thermometer.

What is now the temperature of the boiling water?

As soon as the mercury begins to sputter, take the test-tube away, and remove the water on the mercury with blotting-paper.

From the facts obtained by this experiment, what inference can you draw?

35. Boiling-Point of Water. Men of science throughout the world have agreed to denote by 100° Centigrade the temperature which a thermometer indicates when placed in *steam* from water, which is boiling in an open vessel when the atmospheric pressure is 760^{mm} (that is,

when the atmospheric pressure is sufficient to support a column of mercury 760^{mm} high). Hence, the

Definition. *Water boils at 100° C., when the atmospheric pressure is 760^{mm}.*

Careful experiments have shown that the temperature of water, boiling when the pressure differs but *little* from 760^{mm}, may be found by adding 1° for each 27^{mm} of pressure above 760^{mm}, or by subtracting 1° for each 27^{mm} of pressure below 760^{mm}. Examples like those which follow may make this clearer.

EXAMPLES.

1. When the pressure of the atmosphere is 742^{mm}, the reading, in steam, of a thermometer whose scale was not properly adjusted by the maker is 98°.5. What would be the reading of the instrument for a pressure of 760^{mm}?

Solution. $760 - 742 = 18^{mm}$. As 27^{mm} make a difference of 1°, 18^{mm} would make a difference of $\frac{18}{27}$ of 1°, that is, 0°.67. Hence, the reading of the thermometer, when the pressure is 760^{mm}, would be 98°.5 + 0°.67 = 99°.17.

2. In the example just given, if the pressure had been 778^{mm} and the thermometer reading 100°.4, what would have been the reading for a pressure of 760^{mm}?

DETERMINATION OF FIXED POINTS.

36. Errors of Thermometer Scale. After the mercury has been put into the bulb, and the stem sealed, the next step in making a thermometer is to adjust the scale. In order to adjust the scale, it is necessary to determine the two fixed points. One of the fixed points is determined by putting the bulb and a part of the

stem into melting ice. By putting the thermometer into *steam* coming from boiling water, the other fixed point is found and marked, as was the first one, upon the stem. We have already learned that on the Centigrade scale the freezing-point is marked 0, and the boiling-point 100, and also that the space between the two points is divided into 100 equal parts.

In adjusting the scale, the maker of a cheap thermometer does not use care; hence, frequently, the thermometer does not indicate the true temperature. So careful is a physicist to have his thermometer, though made by a skilled workman, accurate for delicate work, that he always tests the scale for himself. He finds the errors of the thermometer, and in his work makes allowances for them.

Of the two common errors of a thermometer, one arises in marking the position of the freezing-point; the other, in marking the position of the boiling-point.

To test a thermometer for the accuracy of its freezing-point and the accuracy of its boiling-point will be the object of the next experiment.

Experiment 38. *To find whether the fixed points of a thermometer have been properly marked on the scale.*

(For convenience we shall divide this experiment into three parts.)

PART 1. To find the position of the freezing-point.

Apparatus. A thermometer; finely broken ice; a beaker.

Directions. Fill the beaker with clean, finely broken ice. Add enough water to fill the spaces between the lumps of ice. Push the bulb of the thermometer vertically

into the middle of the beaker with its stem vertical until the point marked 0° is only 1^{mm} or 2^{mm} above the surface of the ice, which should be heaped up in the beaker. When the column ceases to fall, record the reading. In this experiment, as well as in other heat experiments, read the thermometer with care. Try to read to tenths of a degree. In reading, be careful to place the eye in such a position that a straight line drawn from the center of the eye would strike the thermometer at right angles at the top of the column.

PART 2. To find the position of the boiling-point.

Apparatus. The thermometer of Part 1; a copper boiler with a copper cone.

Directions. Fill the copper boiler with water to a depth of 3^{cm} or 4^{cm}. Get a cone that will fit tightly, and put it in place on the boiler. Into the side tube leading from the boiler put a cork. Do not stop up the side tube leading from the cone. (Why?) Get a cork stopple that will fit the hole in the top of the cone. In this stopple, with a cork borer, make a hole through which to pass the thermometer. Put the stopple in place in the top of the cone, and through the stopple carefully push the thermometer (Fig. 31) until the point marked 100° is not more than 2^{mm} or 3^{mm} above the top of the stopple. The bulb, however, must not come

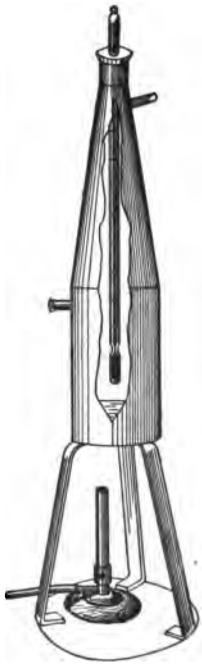


FIG. 31.

within less than 2^{cm} or 3^{cm} of the water in the boiler. Using only one Bunsen burner, keep the water boiling till the mercury column stops rising; then record its position.

We have heated the bulb and a large part of the stem in the *steam*. Now draw up the thermometer till the point marked 0° is just above the stopple. Record the point at which the column becomes stationary.

If this point is different from the one you found when the bulb and a large part of the stem were in the steam, how do you account for the difference?

Suggestion. Consider whether in both cases all the mercury has been heated to the same temperature.

At the time of performing the experiment, record the reading of the barometer.

Why is it necessary to know the reading of the barometer? (See Exps. 36 and 37.)

If the atmospheric pressure had been 760^{mm} when the bulb and the stem were in the steam, find by computation where the boiling-point (100°) would have come on the scale of your thermometer. (See Ex. at end of Art. 35.)

We have already done all the work necessary to find simply whether the freezing-point and the boiling-point have been correctly marked on the scale of the thermometer. Part 3 of the experiment is to bring out another peculiarity of thermometers.

Part 3. To find the position of the freezing-point again.

Apparatus. The same as in Part 1, with a fresh supply of broken ice.

Directions. If the thermometer has just been taken from the steam, let the thermometer remain in the air till the column has fallen to a height of about 40° or 50°. (Why?) Fill the beaker again with clean, finely broken ice, and repeat Part 1. After the column ceases to fall, record its position.

Is the column of mercury shorter or taller than that of Part 1, and how much?

37. Elevation of the Zero-Point; Temporary Lowering of the Zero-Point. When the bulb of a thermometer is put into ice-water, the bulb contracts and a slight elevation of the column is often observed, followed by a fall of the column immediately, or as soon as the mercury in the bulb feels the change in temperature. Even when the thermometer is kept in a place of uniform temperature, careful observers have noted that the bulb of a new thermometer shrinks gradually and perceptibly for some weeks or months. This gradual shrinkage of the bulb raises the zero-point and introduces an error known as the elevation of the zero-point.

The cause of this shrinkage is that the bulb, which was formed by blowing the glass in its plastic state, cooled before its particles had time to regain the relative positions which they had before the pressure necessary to the operation of blowing was applied. As time goes on, these particles, rapidly at first, but after a while more and more gradually, begin to accommodate themselves to one another, and thus produce the shrinkage.

On the other hand, even in good thermometers, the expansion of the bulb, produced by heating it to the

temperature of boiling water, lasts for a little while and produces a *temporary lowering of the zero-point.*

In testing a thermometer, why should the operation of finding the freezing-point come first?

38. Important Precaution in Testing the Boiling-Point of a Thermometer. When testing the boiling-point of your thermometer, you were directed not to allow the bulb to dip into the boiling water; you were to allow the steam, not the water, to come in contact with the bulb. The reasons for this precaution will be brought out in the next experiment and the discussion that comes after it.

Experiment 39. *To find whether salt water will boil at the same temperature as fresh water.*

Apparatus. Two beakers of equal size; a Bunsen burner; a retort-stand and ring; a piece of wire gauze about 15^{cm} square; a thermometer; a spoonful of salt.

Directions. Fill each beaker half full of fresh water. Into one put a spoonful of salt. Set the beakers side by side on the wire gauze over the flame. When the liquids are boiling, record the temperature of each.

When boiling, which liquid has the higher temperature?

In getting the boiling-point of a thermometer, can you give a reason why it is not best to allow the bulb to dip into the water?

NOTE. Besides the possibility of impurities in the water, there is another reason that forbids us to let the thermometer dip into the water. Gay-Lussac found that, for a given atmospheric pressure, water boils at different temperatures in different kinds of vessels. For example, he found the temperature of water boiling in a glass vessel to be higher than

that of water boiling at the same time in a metallic vessel. It was shown, however, by Rudberg that the temperature of the steam which escapes from boiling water is the same in any kind of vessel, and depends only on the pressure at the surface of the water. As the result of a large number of experiments, Rudberg showed that the temperature of steam coming from impure water was the same as that of steam coming from pure water.

EXPANSION OF A SOLID.

39. Cubical Expansion; Linear Expansion. By Exp. 30 we learned that water (a liquid) expands when heated, and by Exp. 32 that air (a mixture of two gases) also expands when heated. When a substance (as the water in the bottle of Exp. 30) expands in all directions, we have what is called *cubical expansion*. In general, all of the solid substances that have been examined expand in all directions as the temperature rises. Often, however, the expansion in length only is observed or measured. This increase in the length is called the *linear expansion* of the solid. No two metals expand the same amount for the same rise of temperature.

The following is an experiment on the measurement of the linear expansion of a solid.

Experiment 40. *To find by what part of its original length a rod of brass increases when its temperature is raised 1°.*

Apparatus. A Bunsen burner; the copper boiler with cone (a sterilizing can makes an excellent steam generator in place of the boiler and cone); two pieces of rubber tube, one about 60^{cm} long, the other about 10^{cm}; dividers; a thermometer; a meter stick; linear expansion apparatus. The linear expansion apparatus consists of a cylindrical tube, called the jacket, which is made of galvanized

iron and is closed at each end with a cork. Through a hole in each cork runs a brass tube whose expansion is to be measured. Pointing at right angles to its length, the jacket has two small side tubes, one near each end and another in the middle. The jacket is set in a horizontal position upon a wooden frame. On the frame is a vertical scale and a pointer. The pointer can be moved along the scale by the expanding rod.

Directions. Lay the jacket (Fig. 32) in its place upon the frame. Rest one end of the brass tube against the screw, at the end of the frame remote from the scale.

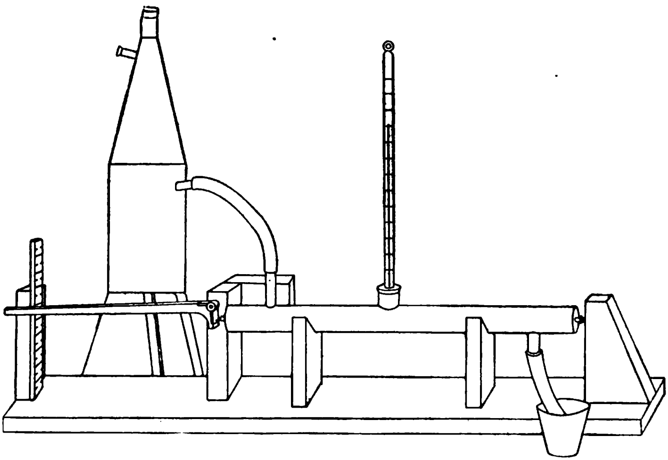


FIG. 32.

The bit of steel wire soldered to the other end of the brass tube should press against the hinge of the pointer. Into the short tube at the middle of the jacket fit a perforated cork through which the thermometer passes. Over the end of the little side tube of the jacket next the pointer slip an end of the long rubber tube; also slip an end of the

short rubber tube over the end of the side tube at the other end of the jacket. Place a beaker under the end of this tube in order to catch the condensed steam that would otherwise drip upon the table. To prevent the jacket from rotating, make the side tube, having the long rubber tube attached, rest against the post that supports the pointer. Measure in centimeters and fractions of a centimeter the long arm of the pointer, that is, from the center of the hinge screw (the screw by which the pointer is pivoted) to the side of the scale facing the cylinder. With a pair of dividers get in centimeters and fractions of a centimeter the length of the short arm of the pointer, that is, from the center of the hinge screw to a point on a level with that where the bit of steel wire touches the hinge. See that the end of the brass rod is resting against the screw at the end of the frame; if it is not, press it back till it touches the screw. Take the reading of the thermometer and also the reading of the pointer on the scale. In taking the reading on the scale, place the eye in such a position that the line of vision just grazes the upper surface of the pointer.

Have the boiler one-third full of water. Put the cone in place and close its top by a stopple, also its side tube. *Do not disturb the frame, pointer, or jacket.* By means of the long rubber tube already attached, join the jacket to the boiler. Heat the water in the boiler and let the steam flow through the jacket until the thermometer column becomes stationary. Record the reading of the thermometer and also the new reading of the pointer. By laying the meter stick beside the jacket, get the length in centimeters of the brass rod in the jacket from the outside

of one stopple to the outside of the other. This measurement is, of course, a very rough one, and may be taken at any stage of the experiment.

From the measurements you have made in this experiment answer the following questions:

(1) What is the distance in centimeters that the pointer moves over in going from its lowest position to its highest on the scale?

(2) How many times does the movement of the pointer multiply the elongation of the brass rod?

SUGGESTION. Divide the length of the long arm by that of the short one.

(3) What is the amount computed (from (1) and (2)) of the elongation of the brass in centimeters?

(4) Through how many degrees of temperature was the brass raised?

(5) What is the average amount of elongation of the brass for 1°?

(6) What is the length of the brass rod in centimeters from the outside of one stopple to the outside of the other?

(7) By what part of its original length (the length given in the answer to (6)) has the brass rod increased for an increase in temperature of 1°?

The answer to (7) is called the *coefficient of linear expansion* of brass, hence the

Definition. *The coefficient of linear expansion tells by what part of its length a body has increased for a rise in temperature of 1°.*

Is the coefficient of linear expansion a quantity or a number?

EXAMPLES.

1. A rod of brass at 15° measures 2 ft. in length; at 95° it measures 2.003 ft. Find the coefficient of linear expansion.

Solution. Increase in length of rod $= 2.003 - 2 \;\; = 0.003$ ft.
Increase in temperature $= \quad 95 - 15 = 80°$.

Increase in length of rod for a rise in temperature of 1° is

$$0.003 \div 80 = 0.0000375.$$

The coefficient of linear expansion $= 0.0000375 \div 2 = .000019$.

The reason for calling this number a coefficient comes from the fact that the amount a given length of brass will expand, when its temperature is raised 1°, can be found by multiplying the given length by 0.000019, that is, we use it just as in algebra, where a coefficient is a numerical value multiplied into a quantity.

2. A bar of lead whose length at 0° was 152.32^{cm} was heated to 100°, when its length was found to be 152.76^{cm}. Find the coefficient of linear expansion of lead.

3. A copper rod that was 15^{m} long at 0° was found to have increased in length by 2.6^{cm}, owing to a rise of 100° in temperature. Find the coefficient of linear expansion of copper.

4. The length of an iron rail at 15° is 30 ft. What will be its length at 10° and at 20°? The coefficient of linear expansion of this iron is $\frac{1}{81900}$.

Solution. $\frac{1}{81900} = 0.0000122.$

To find the length of the rail at 10°: In cooling 1° the rail would shrink by 0.0000122×30 ft., or 0.000366 ft.; but in cooling 5° it would shrink by an amount equal to 5×0.000366 ft., or 0.00183 ft. Hence the length of the rail at 10° would be

$$30 - 0.00183 = 29.99 \text{ ft.}$$

To find the length of the rail at 20°: In having its temperature raised 1°, the rail would expand by 0.0000122×30 ft., or 0.000366 ft.; but in having its temperature raised 5° it would expand by an amount equal to 5×0.000366 ft., or 0.00183 ft. Hence the length of the rail at 20° would be

$$30 + 0.00183 = 30.00183 \text{ ft.}$$

5. What must be the length of a brass rod at 15° in order that at 0° it may be exactly 2^{m} long, the coefficient of linear expansion being 0.000019?

6. Assuming that the maximum temperature of a 30-ft. cast-iron rail exposed to the sun is 50°, and that the temperature of the air at the time of laying the rail is 10°, what must be the minimum distance apart of the adjacent ends of two consecutive rails? The coefficient of linear expansion is 0.000012.

7. The distance by rail from San Francisco to Omaha is 1914 miles. Assuming that the average variation of temperature throughout the year is 50°, what is the variation in the total length of the rails? The coefficient of linear expansion is 0.000012.

EXPANSION OF AIR.

40. Expansion of Air at Constant Pressure; Dalton's Law. In Exp. 32 you found that heat makes air expand. In the experiment just performed, you found by what part of itself a piece of brass will increase in length for a rise in temperature of 1°. In the next experiment you are to find by what part of its *volume* a quantity of air, exposed to a constant (or uniform) pressure, will expand for a rise in temperature of 1°.

Experiment 41. *To find by what part of its volume at 0° an amount of air, at constant pressure, will expand for a rise in temperature of 1°.*

Apparatus. A 250^{cc} flask fitted with a one-hole rubber stopple, having a glass tube 3^{mm} or 4^{mm} in diameter thrust through, reaching nearly to the bottom of the flask and projecting about 2^{cm} above the stopple; about 25^{cm} of pressure tube (to fit the glass tube) and a pinch-cock to close the end of it; ice-water; a Bunsen burner; a large glass jar; a copper boiler.

Directions. Make sure that the flask and all tubes are *dry*. Into the flask insert the stopple with glass tube. Connect the rubber tube with the glass tube. Let the

pinch-cock, as shown in Fig. 33, be placed at the extreme end of the rubber tube farthest from the stopple. (Why?) Have ready warm water in the boiler. Open the cock and plunge the flask beneath the water, but do not get any water into the flask or tubes. (Why?) By laying a piece of wood, with a weight on it, across the boiler, keep the flask submerged. Bring the water to the boiling-point. Boil for five minutes, so that the temperature of the air in the flask may become 100°. Close the pinch-cock and remove the flask from the hot water. Throw a cloth round the flask, as it may collapse. (Why?) When cool, open the pinch-cock under iced water. When as much water as possible has entered, close the cock. Put the flask into the jar filled with ice-water, in which there are many lumps of ice, and keep it submerged five minutes, so that the temperature of the air in the flask may become 0°. Have at hand a beaker containing ice-water, under the surface of which again open the cock. Make the level of the water in the flask and that of the water in the beaker the same, and then close the cock. Remove the flask from the ice-water. Loosen the stopple, raise the rubber tube into a vertical position, and open the cock to let the water run into the flask. (Why?) Get the volume of water in the flask; also the total contents of the flask when the tube is in place.

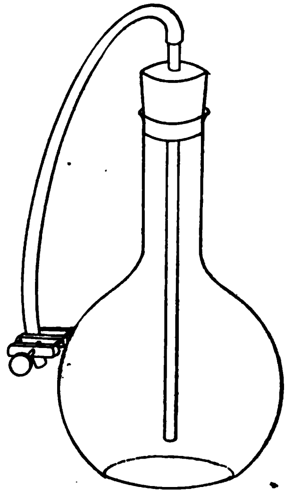


FIG. 33.

From the measurements you have made in this experiment answer the following questions:

(1) What was the volume of air experimented upon at 100°?

(2) What was its volume at 0°?

(3) How much would the volume of air at 0° expand in having its temperature raised 100°?

(4) How much in having its temperature raised 1°?

(5) By what part of its volume at 0° would the air expand for a rise in temperature of 1°?

Reduce the decimal fraction, the answer to (5), to a common fraction with 1 for its numerator.

(For example, the decimal fraction 0.003 is the same as the common fraction $\frac{3}{1000}$. To reduce $\frac{3}{1000}$ to a fraction having 1 for its numerator, divide both numerator and denominator by 3. The result is $\frac{1}{333}$, approximately.)

What is the denominator of the common fraction to which you have reduced the decimal fraction?

At what temperature would 1^{cc} of air, measured at 0°, become 2^{cc}?

At what temperature would 1^{cc} of air, measured at 0°, become 0.5^{cc}?

The relation which connects the different volumes a mass of air may have and the corresponding temperatures, and which enables us to answer questions like those just asked, is called Dalton's Law.

State Dalton's Law.

NOTE. Dalton's Law is also frequently called the Law of Charles. Dalton, in 1801, first published the law. Gay-Lussac, independently of Dalton, published the law in 1802. In his publication, Gay-Lussac states that Charles (1746–1823), Professor of Physics at Paris, had in 1787 noted the law, but had never published it.

41. The Air Thermometer; the Absolute Zero. Careful experiments have shown that not only air but all gases also *expand by* $\frac{1}{273}$, *or 0.00366, of their volume at 0° for each degree in increase in temperature, and contract by a like amount (of their volume at 0°) for each degree in fall of temperature.* This regularity in the expansion and contraction of gases has led to the construction of the *air thermometer*, in which are observed the changes in volume of a quantity of air so confined that, no matter how much it may expand or contract, its pressure will always be constant. While we shall not describe the construction of an actual *air thermometer*, an instrument cumbersome and, in practice, difficult to use (in fact, it is used only occasionally as a standard with which to compare mercury thermometers that are for use in delicate work), we shall describe an *ideal* instrument which will make clear the action of the *air thermometer* in practice.

This *ideal* instrument (Fig. 34) consists of a long horizontal glass tube of uniform bore, closed at one end and graduated in equal parts. It contains a quantity of dry

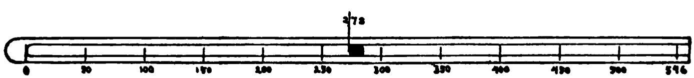


FIG. 34.

air which, at 0°, occupies 273 of these parts, and which is cut off from the external air by a small pellet of mercury. As a quantity of air at 0°, when heated 1°, expands by $\frac{1}{273}$ of its volume, it follows that if the air in the tube has its temperature raised 1°, the volume which it will occupy will be 274 of these parts. Had the temperature

been raised to 10°, the volume of the air would have been 283 of these parts. As the temperature rises, the air will occupy more and more of these parts. On the other hand, had the air in the tube been cooled to —1°, the volume of air would have occupied 272 parts; if cooled to —10°, the volume occupied would have been 263 parts. As the cooling continues, the volume of air goes on shrinking; and Dalton's Law [*The volume of a gas measured at 0° increases (or diminishes) by* $\frac{1}{273}$ *of itself for every degree that the temperature increases (or diminishes).*] has been found to hold for the lowest temperatures that we have obtained. So low a temperature as —273° has never been reached, but, theoretically, the volume occupied by a gas at this extremely low temperature would be *no space at all.* (In all probability a gas, when cooled, would become a liquid long before the temperature fell as far as — 273°; should the gas have then become a liquid, Dalton's Law of course would no longer apply.)

The point — 273° is called the *absolute zero of the air thermometer*, and it is extremely convenient, in dealing with questions relating to gases, to reckon temperatures not from the freezing-point, but from the absolute zero. The closed end of the tube is marked 0°, and the freezing-point, which is marked 0° on the Centigrade scale, would be marked 273° on this absolute scale. (Why?) To get the absolute temperature in Centigrade degrees, we have only to add 273° to the reading of the Centigrade scale.

QUESTIONS. What is the absolute temperature, when the temperature on the Centigrade scale is 0°? 10°? 100°? 273°? —10°? —100°? —273°?

Dalton's Law might be stated thus: *The volume of a gas varies directly as its temperature on the absolute scale.*

EXAMPLES.

1. If the volume of a certain amount of gas is 500^{cc} at $50°$, what would be its volume at $150°$?

Solution. By Dalton's Law in the form just stated, we have

$$50 + 273 : 150 + 273 = 500 : x,$$

or
$$323 : 423 = 500 : x,$$

whence
$$323x = 211500,$$
$$\therefore x = 654.8^{cc}.$$

2. A quantity of oxygen occupies 150^{cc} at $15°$. What space will it occupy if the temperature is reduced to $0°$?

3. A gas has its temperature raised from $8°$ to $72°$; at the latter temperature it occupies 12 liters. What was its original volume?

NOTE. 1 liter $= 1000^{cc}$.

4. At $0°$, the space occupied by 1.293^{g} of air is 1 liter. At what temperature will the weight of 1 liter of air be 1^{g}?

42. Problems involving both Boyle's Law and Dalton's Law. When the pressure changes and the temperature does not, Boyle's Law gives us the means of finding the volume of a gas; on the other hand, when the temperature changes and the pressure does not, Dalton's Law gives us the means of finding the volume of the gas.

There is a class of problems in which we are required to find the volume after the gas has undergone a change both in pressure and in temperature. In solving such problems, first find the volume that would result from a change in pressure, and then find what this volume would become when the temperature is changed. In the following set of examples, both Boyle's Law and Dalton's Law are involved.

EXAMPLES.

1. A gas of volume 304^{cc}, temperature $127°$, and pressure 75^{cm} has its pressure raised to 76^{cm} and its temperature lowered to $27°$. Find its volume after these changes.

Solution. Let x denote the required volume, then collecting what is given in the problem, we have

V	P	t
304	75	127
x	76	27

Supposing the temperature to remain constant, let us denote by y the volume of the gas after the pressure has changed from 75^{cm} to 76^{cm}. By Boyle's Law, we have

$$304 : y = 76 : 75,$$
$$76\,y = 22800,$$
$$\therefore y = 300^{cc}.$$

Now find the volume, x, which the volume 300^{cc} becomes after the temperature has changed from $127°$ to $27°$. By Dalton's Law, we have

$$300 : x = 127 + 273 : 27 + 273,$$
$$300 : x = 400 : 300,$$
$$400\,x = 90000,$$
$$x = 225^{cc}.$$

Hence, the volume which results from the change in pressure and the change in temperature is 225^{cc}.

2. The volume of a certain quantity of air at $27°$, and under a pressure of 75^{cm}, is 1000^{cc}. What would be its volume at $127°$ under a pressure of 150^{cm}?

3. If a mass of air has a volume of 140^{cc} when the temperature is $136°.5$ and the pressure is 57^{cm}, what will the volume become when the temperature is $0°$ and the pressure is 76^{cm}?

4. If a mass of air has a volume of 300^{cc} when the temperature is $0°$ and the pressure 76^{cm}, what will the volume become when the temperature is $91°$ and the pressure is 95^{cm}?

5. When the temperature is $99°$ and the pressure is 95^{cm}, a mass of gas has a volume of 320^{cc}. What must be the temperature in order that the gas may have a volume of 300^{cc}, when the pressure is 76^{cm}?

6. If a mass of gas has a volume V_1 when the pressure is P_1 and the temperature is t_1, and a volume V_2 when the pressure is P_2 and the temperature is t_2, show that

$$\frac{V_1 P_1}{T_1} = \frac{V_2 P_2}{T_2}$$

where $T_1 = t_1 + 273$, and $T_2 = t_2 + 273$.

TRANSMISSION OF HEAT.

43. Conduction, Convection, and Radiation of Heat. Heat is communicated from one body to another by one or more of the three ways, *conduction*, *convection*, and *radiation*. It will be the object of the next five experiments to make you acquainted with these three ways by which heat is communicated.

Experiment 42. *To find whether wood or copper is the better conductor of heat.*

Apparatus. A match; a piece of copper wire of the same size as the match; a Bunsen burner.

Directions. Between the thumb and the forefinger of one hand hold the match by its end; in the same manner with the other hand hold the copper wire. In the tip of the flame of the Bunsen burner thrust together the end of the match and the end of the wire.

Which can you hold the longer?

Which is a good conductor of heat?

Which is a poor conductor of heat?

Experiment 43. *To find whether copper or iron is the better conductor of heat.*

Apparatus. A Bunsen burner; a copper wire and an iron wire, each about 10cm long and as nearly as possible of the same cross-section.

Directions. Hold one wire in each hand so that the ends shall be together in the top of the flame.

Which is the better conductor?

What is the reason for your answer?

Why does a piece of oil-cloth feel cold when you step upon it with the bare feet, while a piece of woolen carpet feels warm?

Why are wooden handles put on soldering-irons?

Experiment 44. *To find what is meant by convection of heat.*

Apparatus. A Bunsen burner.

Directions. Hold your open hand palm downwards as high above the flame of the Bunsen burner as you can, and lower it gradually. Estimate or measure roughly the distance above the flame at which the temperature becomes unbearable.

The gases produced by the chemical action in the flame are hot and expand. The cooler air pushes them up. They strike the hand and warm it.

Experiment 45. *To find whether water is heated by convection, when heat is applied beneath.*

Apparatus. A Bunsen burner; a retort-stand and ring; a piece of wire gauze about 15^{cm} square; a beaker; fine sawdust or bran.

Directions. Into the beaker half full of water sprinkle a little pinch of sawdust, then place the beaker on the gauze and heat it over the flame.

By watching the movements of the sawdust, should you infer that the water is heated by convection?

What peculiarity in the movements of the sawdust would lead you to the conclusion that the water in this experiment is heated by convection?

Experiment 46. *To find what is meant by radiation of heat.*

Apparatus. A Bunsen burner.

Directions. In Exp. 44 you found how close the hand, when held above, could approach the flame. At this same distance from the side of the flame hold the hand and gradually bring it nearer and nearer.

How close can the hand approach?

Why is not the heat which reaches your hand brought to it by convection?

Why is not the heat conducted to your hand?

Heat that passes through a medium without warming it, but is capable of warming an object even at a great distance from the source of heat, is called *radiated heat.*

The heat which we receive from the sun reaches our earth through the immense space between the sun and the earth by *radiation.*

Point out how the flame of the Bunsen burner illustrates the three modes (convection, conduction, and radiation) by which heat may pass from one point to another.

By how many of the three modes by which heat is transmitted is your school building warmed?

In what way do the glowing coals in an open grate give heat to a room?

Why is an open grate a good means of ventilation?

CALORIMETRY.

44. Measurements of Quantities of Heat. In the measurements we have made thus far, we have used as our unit of length the centimeter; as our unit of weight, the gram. In measuring quantities of heat, we shall use a special unit. This unit of heat is named the calorie (pronounced *căl'ŏ rie*).

Definition. *The calorie (the unit of heat) is the amount of heat required to raise the temperature of 1^g of water 1°.*

The beginner often asks the question, "How much heat is there in a calorie?" Remember that the calorie is a unit of heat, just as the centimeter is a unit of length.

Experiment 47. *To find whether the quantity of heat given up by a known weight of water in falling through a certain range of temperature in one part of the thermometric scale, is able to raise the same weight of water through the same number of degrees in a different part of the scale.*

Apparatus. Two nickel-plated cups; a copper boiler; a Bunsen burner; a thermometer; a 100^{cc} graduate.

Directions. Into one of the cups pour 175^g of cold water (temperature 5° to 10°), into the other pour 175^g of warm water (temperature 50° to 60°). Stir the cold water with the thermometer and note its temperature; then, stirring as before, immediately take the temperature of the warm water. Before the warm water has had time to cool below the temperature you have noted, pour it all into the cold water, stir the mixture well with the thermometer, and take the temperature.

Why should a liquid be thoroughly stirred before taking its temperature?

How many degrees has the temperature of the hot water fallen?

How many degrees has the temperature of the cold water risen?

In what way (conduction, convection, or radiation) does the cold water receive heat before it is mixed with the warm?

In what way does the warm water, before mixing, lose heat?

This experiment leads to the following definition:

Definition. *The calorie is the amount of heat which 1g of water gives up in cooling 1°.*

Experiment 48. *To find the temperature resulting from mixing equal weights of water and mercury of different temperatures.*

Apparatus. A copper boiler; Bunsen burner; iron pan; thermometer; a 100cc graduate; three beakers of nearly equal size; broken ice.

Directions. In the graduate, placed in the pan, measure out 100g of mercury (sp. gr. = 13.6). Pour the mercury into the beaker, and put the beaker into the water in the copper boiler with the lighted burner beneath. Keep the mercury dry.

Make ready in a beaker 100g of water 10° colder than the air of the room. (Why?) Have the mercury 10° warmer than the air of the room. (Why?) Then pour both, the water first, into the third beaker, which should have the same temperature as the air of the room, the

temperature that it will have, of course, if the beaker has been standing in the room for a little while. With the thermometer, stir the two liquids thoroughly, for about half a minute. Note the temperature of the mixture.

Which liquid appears to have had the greater influence in producing the final temperature?

Using the same weights of water and dry mercury as before, repeat the experiment; only have the temperature of the mercury 10° below the temperature of the air of the room, and the water 10° above the temperature of the air of the room.

In this case which liquid has the greater effect in producing the final temperature?

Would it take as much heat to warm 1^g of mercury 1° as to warm 1^g of water 1°?

45. Thermal Capacity; Calorimetry; Method of Mixtures. The *thermal capacity* of a body (solid, liquid, or gaseous) is the number of calories (see Def., page 100) necessary to raise its temperature 1°. The process of measuring quantities of heat, for example, the thermal capacity of a body, is called *calorimetry*. One of the methods of calorimetry, called the *method of mixtures*, is very common, and is that which you will employ in your work in calorimetry. The method of mixtures consists essentially in putting the body to be tested, after its weight and temperature have been noted, into a quantity of water of known weight, whose temperature is known but is different from that of the body. The resulting common temperature, known as the *temperature of the mixture*, is noted.

The body may be a solid, so that it and the water could not be literally *mixed.* It is customary, however, to refer to a solid and the water, when put together in one vessel, as the *mixture.*

Experiment 49. *To find, by the method of mixtures, the amount of heat given out by 1^g of lead in cooling $1°$.*

Apparatus. Lead shot; a copper dipper to heat the shot in; a copper boiler; a thermometer; a calorimeter (this is a tall cup of nickle-plated brass, brightly polished); a platform balance; a little ice-water; a 100^{cc} graduate; a piece of cardboard to cover the dipper.

Directions. Into the dipper put 500^g of shot. Cover the top of the dipper with the cardboard, in order to keep the shot from being cooled by air currents. Through a hole in the cardboard pass the thermometer and plunge its bulb into the shot. In the boiler place the dipper with its flange resting on the top (Fig. 35). In order that there may be no danger of boiling the water in the boiler all away, have the water reach nearly to the bottom of the dipper at the start. Close the side tube of the boiler with a stopple, so that the steam will have to pass out under the flange of the dipper.

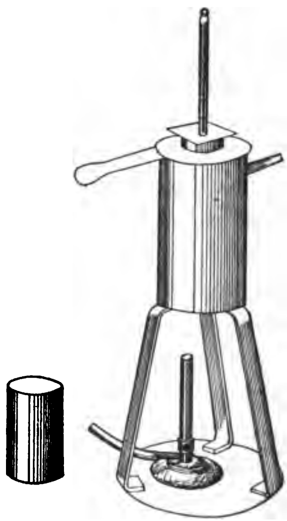


FIG. 35.

While the shot are heating, wipe and weigh the calorimeter (pronounced *calo-rim'e-ter*), and measure out 100^g of water whose temperature is 6° or 8° colder than the air of the room, that is, 6° or 8° below the temperature as indicated by a thermometer near the spot where the experiment is to be performed. Pour the water into the calorimeter, and place it conveniently near the heating apparatus, but shielded from the heat.

When the temperature of the shot has become practically constant, that is, about 100°, remove the thermometer, and allow it to cool till the column stands at about 50° or 40°. Then put the thermometer into the calorimeter, stir the water and give the thermometer time to come to the temperature of the water. Record the temperature of the water. Meanwhile, stir the shot frequently with an iron rod or any convenient object. Then take the thermometer out of the water. Take the dipper from the boiler, remove the cardboard (no time should be lost in this part of the experiment, for the dipper and its contents after being taken from the boiler immediately begin to cool), taking care to spill neither shot nor water, very quickly pour the shot into the calorimeter. (This can be successfully done by the student, if he has practised beforehand pouring some shot from a dipper into an empty calorimeter.) At once stir the mixture of shot and water quickly and thoroughly with the thermometer. Record the reading of the thermometer as soon as the shot and water have reached a common temperature, which will probably be a few degrees above the temperature of the room. It is of importance to get this temperature accurately. With vigorous stirring it takes but

a few seconds for the water and the lead to reach a common temperature. Glance at the thermometer column frequently; after the first somewhat violent fluctuations the mercury will become stationary for an instant and then begin to fall slowly; the reading of the column, at the instant when it becomes stationary, is the one to be noted.

From the data obtained answer the following questions:

(1) If x denotes the amount of heat given out by 1^g of lead in falling 1°, how much heat would be given out by the whole mass of lead in falling 1°?

(2) How much heat would be given out by the whole mass of lead in falling from the temperature it had when it entered the water to the temperature of the mixture?

(The heat given out by the lead warmed the calorimeter and the water in it. Both the water and the calorimeter had the same temperature.)

(3) How much heat does it take to raise 1^g of water 1°? (See Def., page 100.)

(4) How much heat does it take to raise the whole mass of water 1°?

(5) How much heat does it take to raise the whole mass of water from the temperature it had when the shot were poured in to the temperature of the mixture?

(6) If it takes 0.1 of a calorie to raise 1^g of the substance of which the calorimeter is made 1°, how much heat will it take to raise the temperature of the whole calorimeter 1°?

(7) How much heat will it take to raise the whole calorimeter from its temperature when the shot entered to the temperature of the mixture?

When you have answered the questions, make an equation. On one side put the expression [the answer to (2)] for the quantity of heat given out by the lead in cooling; on the other, the quantity of heat received by the water [the answer to (5)] plus the quantity of heat received by the calorimeter [the answer to (7)]. On solving this equation, you will get the amount of heat given out by 1^g of lead in cooling $1°$.

A special name is given to the numerical value of the amount of heat yielded by 1^g of a substance when its temperature falls $1°$. This value is called the *specific heat* of the substance. The amount of heat necessary to raise 1^g of a substance $1°$ is equal to the amount of heat given out by 1^g of the substance when its temperature falls $1°$; hence,

Definition. *The specific heat of a substance is the numerical value of the thermal capacity of 1^g of the substance.*

NOTE. The answer to (6) is called the *water equivalent* of the calorimeter. Why?

QUESTIONS. What is the specific heat of water? of the calorimeter? What was the thermal capacity of the lead used in the preceding experiment? of the water? of the calorimeter?

EXAMPLES.

1. Into 110^g of water at 15°, contained in a vessel the thermal capacity of which is equal to that of 10^g of water, are put 200^g of a certain solid at 100°, and the resulting temperature of the whole is 25°. Compute the specific heat of the solid.

Solution. Let x denote the specific heat of the solid.

Then $200\,(100 - 25)\,x =$ the number of calories given out by the lead in cooling from 100° to 25°.

$110\ (25 - 15)$ = the number of calories received by the water in having its temperature raised from 15° to 25°.

$10\ (25 - 15)$ = the number of calories received by the calorimeter in having its temperature raised from 15° to 25°.

Now the number of calories given out by the lead were received by the water and the calorimeter, so we can add together the calories received by the water and the calorimeter and form an equation by putting this sum equal to the number of calories given out by the lead; that is,

$$200\ (100 - 25)\, x = 110\ (25 - 15) + 10\ (25 - 15).$$

Solving this equation, we have $x = 0.08$.

2. A coil of copper wire weighing 45.1g was dropped into a calorimeter containing 52.5g of water at 10°. The copper before immersion was at 99°.6, and the common temperature of copper and water after immersion was 16°.8. Find the specific heat of the copper wire.

NOTE. When nothing is said about the weight of the calorimeter, as in the example just given, make no account of the calorimeter in the computation.

3. Find the specific heat of a substance 100g of which at 90°, when immersed in 250g of water at 12°, gives a resulting temperature of 18°.

4. Compute the specific heat of silver from the following data:

Weight of silver	= 10.2g
Weight of water	= 84g
Temperature of silver	= 102°
Initial temperature of water	= 11°.08
Temperature of mixture	= 11°.69

5. If the specific heat of mercury is 0.0333, what will be the temperature of 100g of water taken at 0°, into which 1000g of mercury at 100° are poured and thoroughly stirred?

Solution. Let $t°$ denote the required temperature.

Then $1000\ (100 - t)\ 0.0333$ = number of calories of heat given out by mercury in cooling from 100° to $t°$.

$100\ (t - 0)$ = number of calories of heat received by the water in having its temperature raised from 0° to $t°$.

These two quantities of heat are equal, hence we form the equation

$$1000\ (100 - t)\ 0.0333 = 100\ (t - 0).$$

On solving this equation, $t = 25°$ (nearly).

6. Equal volumes of turpentine at 70° and of alcohol at 10° are mixed; find the resulting temperature. (Specific gravity of turpentine, 0.87; of alcohol, 0.80. Specific heat of turpentine, 0.47; of alcohol, 0.62.)

LATENT HEAT.

46. Change of State. Every substance with which we are acquainted must be in one of the three states, or forms, the solid, the liquid, or the gaseous. Sometimes we find a substance in the solid state, sometimes in the liquid state, and sometimes in the gaseous. For example, water is a substance with which we are acquainted in the solid state (as ice), in the liquid state, and in the gaseous state (as steam). When a solid, such as ice, melts, it passes from the solid state into the liquid. The solid is said to have undergone a *change of state.* On the other hand, when a liquid, like water, freezes, it has passed from the liquid to the solid state. In this case also a change of state has taken place. When a liquid, like water, passes into the gaseous form, as steam, or when the gas condenses, or passes back to a liquid, there is also a change of state. In brief, then, when a substance passes from one of these states to another, a change of state is said to have taken place.

The next three experiments deal with change of state.

Experiment 50. *To find what changes in temperature are produced by applying heat to equal weights of ice and ice-water, contained in separate vessels.*

Apparatus. Two similar beakers; a small pan about 3 inches deep; a thermometer; a Bunsen burner; a retort-stand and ring; ice; a platform balance.

Directions. Fill one beaker two-thirds full of small pieces of ice, and put into the other an equal *weight* of ice-water. In the pan of boiling water that you have just removed from the flame, put both beakers, after noting the temperature of each. Without putting the pan over the flame again, stir the ice almost constantly, and occasionally stir the water in the other beaker. Let the beakers stand in the hot water till nearly all the ice has melted, then quickly note the temperature of each.

Both beakers are of the same size and have stood in the hot water for the same length of time.

In which one does the temperature remain unchanged, or almost unchanged?

As both beakers were at the same or nearly the same temperature at the start, what has become of the heat that went to the beaker in which the temperature rose the least?

In what state were the contents of this beaker before it was put into the hot water?

In what state were the contents after it had been subjected to heat?

Definition. *The heat required by substances for changing their state is called latent heat.*

A better name is *heat of fusion*, when the change is from solid to liquid; *heat of vaporization*, when the change is from liquid to vapor. Heat of fusion is sometimes called *latent heat of melting.*

Definition. *The heat that is used in raising the temperature of a body is called sensible heat, as it can be perceived by the sense of touch.*

Experiment 51. *To find the amount of heat necessary to change 1^g of ice whose temperature is $0°$ into 1^g of water whose temperature is $0°$.*

Apparatus. A calorimeter; a platform balance; a thermometer; a piece of cardboard to cover calorimeter; clear ice.

Directions. Record the weight of the calorimeter. Put into the calorimeter 200^g of water whose temperature is about 55°. Cover the calorimeter with the piece of cardboard, having a notch at one side to admit the thermometer. (The cardboard retards the evaporation of the water.) Take a lump of clear ice weighing from 140^g to 150^g, which should have been selected before the warm water was put into the calorimeter, and in an ice-cutting machine or in a cold box break it up quickly into pieces about as large as chestnuts. Use neither snow nor snow-ice. Record the temperature of the water after stirring, and immediately, avoiding the wetter portions, put into the calorimeter about 100^g of the ice. Do not weigh out 100^g of crushed ice, but estimate the amount as nearly as you can. Stir thoroughly, though not violently, with the thermometer. As the last particles of ice melt, record the temperature indicated by the thermometer. If so much ice has been put in as to cool the water below 5°, dip out the ice which remains unmelted, taking out as little water as possible. Weigh the calorimeter and contents to find the weight of ice melted.

By means of the data obtained answer the following questions, which indicate the course of reasoning that must be gone through in order to find the number of calories necessary to melt 1^g of ice without changing its temperature.

(1) If x denotes the number of calories required to melt 1^g of ice, how many calories are necessary to melt the whole weight of ice?

(2) How much heat is required to raise the temperature of 1^g of water $1°$? (See Def., page 100.)

(3) How many calories are required to raise the temperature of the liquefied ice (ice-water) from $0°$ to the temperature of the mixture?

(4) How much heat is given out by 1^g of water in cooling $1°$? (See Def., page 101.)

(5) How many calories are given out by the warm water in cooling to the temperature of the mixture?

(6) In cooling $1°$, what part of a calorie is given out by 1^g of the substance of which the calorimeter is made?

(The specific heat of the calorimeter is 0.1.)

(7) How many calories are given out by the whole calorimeter in cooling $1°$?

(8) How many calories are given out by the calorimeter in cooling from its first temperature to the temperature of the mixture?

(The first temperature of the calorimeter is the same as that of the water in it at the start.)

Make an equation, putting on one side the quantity of heat necessary to melt the ice [answer to (1)] plus the quantity of heat necessary to raise the ice-water to the temperature of the mixture [answer to (3)]; on the other,

put the quantity of heat given out by the water in cooling to the temperature of the mixture [answer to (5)] plus the quantity of heat given out by the calorimeter in cooling to the same temperature [answer to (8)]. On solving this equation you will get the quantity of heat necessary to melt 1^g of ice.

Definition. *The latent heat of fusion of a substance is the number of units of heat required to change 1^g of the substance at its melting-point into liquid at the same temperature.*

How many units of heat are required to melt 1^g of ice?

How many units of heat will be given out by 1^g of ice-water in freezing?

Which takes the more heat, to melt 1^g of ice, or to raise the temperature of 1^g of water from 0° to 100°?

EXAMPLES.

1. From the following experimental data find the latent heat of water:

Weight of water	$= 200^g$.
Temperature of water	$= 58°$.
Weight of ice	$= 118^g$.
Temperature of mixture when all the ice was melted	$= 8°$.

(Latent heat of water is another name for latent heat of melting.)

Solution. Let x denote the number of calories necessary to melt 1^g of ice.

Then $118\,x$ = number of calories necessary to melt the whole weight of ice.

$118\ (8 - 0)$ = number of calories necessary to raise the liquefied ice from 0° to 8°.

$200\ (58 - 8)$ = number of calories given out by the warm water in cooling from 58° to 8°.

Now the number of calories received by the ice in melting and by the liquefied ice in having its temperature raised from 0° to 8° was given out by the warm water when its temperature fell from 58° to 8°; in other words, the heat given out by the warm water in cooling went to melt the ice and to raise the temperature of the liquefied ice, so we have the equation

$$118\,x + 118\,(8 - 0) = 200\,(58 - 8).$$
$$\therefore x = 76.7 \text{ calories.}$$

2. As the result of experiment, it is found that 25^g of copper at the temperature of 100° are just sufficient to melt 2.875^g of ice at 0°, so that the water and the copper are finally at 0°. Taking 80 calories as the latent heat of water, from these data find the specific heat of copper.

3. What quantity of water at 15° will be required to melt 1000^g of ice at 0°, so that the resulting mixture shall be at 5°? (Take the latent heat of water as 80.)

Experiment 52. *To find the number of calories 1^g of steam, whose temperature is 100°, gives up in changing into water at 100°.*

Apparatus. A calorimeter; a thermometer; a "trap" (consisting of a side-necked test-tube with a perforated stopple, through which, and reaching nearly to the bottom of the test-tube, a piece of glass tube is thrust); a copper boiler with a cone (or better a sterilizing can); a Bunsen burner; a piece of cardboard to cover the calorimeter (this cardboard should have a notch at one edge to admit the tube conducting the steam, and in the middle a perforation to admit the thermometer); two pieces of rubber tube, one about 50^{cm} in length, the other about 10^{cm}.

NOTE. The object of the trap is to catch the water from the steam that is condensed, when it passes from the boiler through the long tube; the steam must pass through a long tube in order to keep the calorimeter a long distance from the Bunsen burner, otherwise the contents of the calorimeter would have its temperature raised by the heat of the flame.

Directions. Fill the boiler one-third full of water. Fit a cone tightly on the boiler (Fig. 36). Stop the opening in the apex of the cone and also the side tube near

the apex. Over the side tube of the boiler slip the longer piece of rubber tube. Slip its other end over the end of the glass tube that reaches to the middle of the test-tube. Over the side tube of the test-tube slip the short piece of rubber tube with a bit of glass tube in one end.

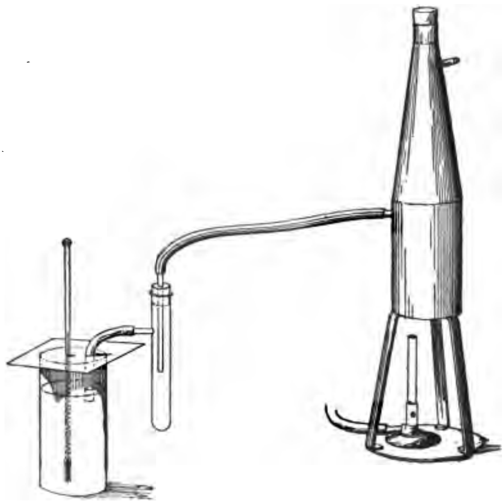


FIG. 36.

Record the weight of the calorimeter. Then weigh carefully in the calorimeter 275^g of water whose temperature is about 15° below the temperature of the room. When the steam flows strongly through the trap, the mouth of the rubber tube, attached to the side tube, should be plunged beneath the surface of the water in the calorimeter, but not so deep as to prevent the steam from condensing with a very audible sound. Cover the calorimeter with the cardboard, and through the opening pass the

thermometer. Frequently stir the water with the thermometer. Screen the calorimeter from the heat radiated by the flame and other hot objects. When, by the condensation of the steam, the temperature of the water has been raised about 15° above the temperature of the room, remove the tube from the water. Stir the water thoroughly with the thermometer and take its temperature.

What was the object in having the final temperature of the water as much above the temperature of the room as the initial temperature of the water was below it?

Weigh the calorimeter and contents, and thus find the weight of the condensed steam. This weighing is very important and should be carefully done.

Notice that there are two portions of heat yielded to the cold water and to the calorimeter; one coming from the steam during the act of condensation, the other coming from the condensed steam (water at 100°) while cooling to the final temperature.

In the computation of the latent heat of vaporization, take 0.1 as the specific heat of the substance of which the calorimeter is made.

(1) If x denotes the number of calories given up by 1^g of steam at 100° in changing into water at 100°, how many calories are given up by the whole weight of the steam during this change?

(2) How many calories are given up by 1^g of water in cooling 1°? (See Def., page 101.)

(3) How many calories are given up by the condensed steam (water at 100°) in cooling 1°?

(4) How many calories are given up by the condensed steam in cooling to the temperature of the mixture?

(5) How many calories are required to raise the temperature of 1g of water 1°? (See Def., page 100.)

(6) How many calories are required to raise the whole weight of cold water from its first temperature to the temperature of the mixture?

(7) How many calories are required to raise 1° the temperature of 1g of the substance of which the calorimeter is made?

(8) How many calories are required to raise the temperature of the whole calorimeter 1°?

(9) How many calories are required to raise the calorimeter from its first temperature (the temperature of the water at first) to the temperature of the mixture?

Make an equation, putting on one side the quantity of heat given up by the steam in changing into water [answer to (1)] plus the quantity of heat given up by the condensed steam in cooling to the temperature of the mixture [answer to (4)]; on the other, the quantity of heat necessary to raise the cold water from its first temperature to that of the mixture [answer to (6)] plus the quantity of heat required to raise the calorimeter from its first temperature to the temperature of the mixture [answer to (9)]. On solving this equation you will get the quantity of heat given up by 1g of steam in changing into water.

How many calories would be required to change 1g of water whose temperature is 100° into steam whose temperature is 100°?

Which would give out the greater amount of heat, 1g of steam whose temperature is 100° in changing into water, or 1g of water in cooling from 100° to 0°?

QUESTIONS AND EXAMPLES.

1. Why is steam such an effective agent in heating buildings?

SUGGESTION. Consider the amount of heat given out by steam in condensing.

2. Beginning with the fire beneath the steam boiler, describe as fully as you can the transferences of heat that occur in the process of heating a room by means of coils of steam-pipe.

3. Using as a model the definition of the *latent heat of fusion*, page 112, state a definition for the *latent heat of vaporization*.

4. From the results of the following experiment, allowing for the heat absorbed by the brass calorimeter, compute the latent heat of steam (the latent heat of vaporization):

Weight of calorimeter	$= 326.3^g$.
Weight of calorimeter and water	$= 757.7^g$.
Weight of steam condensed	$= 46.35^g$.
Temperature of steam	$= 100°$.
Temperature of water before experiment	$= 7°.5$.
Temperature of water after experiment	$= 62°.5$.
Specific heat of brass	$= 0.09$.

5. A vessel containing 30^g of ice is placed over a Bunsen burner; how many calories will be required to melt the ice and to vaporize the water completely? (Latent heat of fusion, 80; latent heat of vaporization, 536.)

47. Freezing-Mixtures. When ice melts, heat disappears. The form of the ice is changed from the solid to the liquid, but there is no marked rise in temperature till all the ice has vanished. The heat that has disappeared in this process has gone to change the *form* of the substance, without appreciably raising the temperature of the water as long as any of the ice remains. When ice melts, heat passes from surrounding objects that have a higher temperature than that of the ice to the ice itself; consequently, those objects which in this manner part with

heat must become colder, unless heat is constantly supplied to them. When salt is mixed with ice the ice melts more quickly and so takes heat more rapidly from surrounding objects. A mixture of ice and salt is called a *freezing-mixture*. When this mixture surrounds a freezer full of cream, the cream freezes. The mixture in becoming liquid has taken heat from the cream, and the cream has frozen. Is the freezing-mixture colder than the cream? This is a pertinent question which the following experiment will enable you to answer.

Experiment 53. *To find whether the temperature of a mixture of melting ice and salt is different from that of melting ice.*

Apparatus. A beaker; a thermometer; a platform balance; snow or broken ice; salt.

Directions. Mix in the beaker one part by weight of salt and two parts by weight of snow or broken ice. Put a thermometer into the mixture, and note the temperature.

What conclusion do you reach?

THERMOMETRIC SCALES.

48. Centigrade, Fahrenheit, and Reaumur Scales. Besides the Centigrade scale, with which you have become familiar, there are two others: the Fahrenheit, in use in England and the United States, and the Reaumur, in use in Russia and a few places in the north of Germany. The Fahrenheit scale, named for Fahrenheit, of Dantzig in Germany, who about 1714 was the first to construct thermometers whose readings were the same when put

into substances having the same temperature, has for its zero-point the temperature of a mixture of snow and salt. On this scale the freezing-point is 32° and the boiling-point 212°. On the Reaumur scale the freezing-point is marked 0, just as on the Centigrade scale, but the boiling-point is marked 80.

Occasionally you will be required, when given the reading of one of these scales, to find the corresponding reading of one of the others. The following table will help you to understand how this can be done.

	C.	F.	R.
Freezing-point	0	32	0
Boiling-point	100	212	80

C. stands for Centigrade, F. for Fahrenheit, and R. for Reaumur.

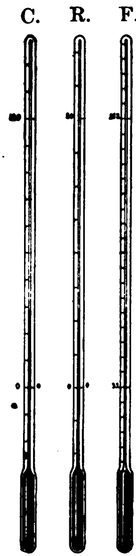


FIG. 37.

Between the fixed points on the Centigrade scale (Fig. 37) there are 100 equal parts; on the Fahrenheit, 180; on the Reaumur, 80. Hence a change of 5° C. is equivalent to a change of 9° F., or 1° C. is equal to $\frac{9}{5}$° F. Fahrenheit's zero is 32° below the freezing-point. Hence, to reduce Fahrenheit readings to Centigrade readings subtract 32 from the number of Fahrenheit degrees and multiply the remainder by $\frac{5}{9}$.

$$C = \tfrac{5}{9}\,(F - 32).$$

To reduce Centigrade readings to Fahrenheit, multiply the number of Centigrade degrees by $\frac{9}{5}$ and add 32.

$$F = \tfrac{9}{5}\,C + 32.$$

EXAMPLES.

1. Show by a process of reasoning similar to that just given that

$$R = \tfrac{4}{9}(F - 32)$$
$$F = \tfrac{9}{4}R + 32$$

2. Show that

$$C = \tfrac{5}{4}R$$
$$R = \tfrac{4}{5}C$$

3. Find the corresponding temperatures on the Centigrade scale of the following melting points:

Lead, 626° F.; bismuth, 508° F.; tin, 442° F.; Rose's metal, 200° F.

NOTE. Rose's metal is an alloy of the first three.

4. Find the equivalents on the Reaumur scale of the following temperatures:

Usual temperature of the human body = 98°.4 F.
" " " a common frog = 64° "
" " " a chicken = 111° "

5. In the expedition to China in 1829 the Russian army experienced for several days a temperature of −32°.8 R. What would this be on the Fahrenheit scale?

6. The absolute zero[1] is −273° C. What is this on the Fahrenheit scale?

7. At what temperature is the reading of the Centigrade identical with that of the Fahrenheit?

FORMATION AND CONDENSATION OF VAPOR.

49. Evaporation; Boiling. The object of the next two experiments is to bring out the distinction between *evaporation* and *boiling*.

Experiment 54. *To find whether vapor will form quietly at the surface of water.*

Apparatus. A large test-tube; a Bunsen burner; a small piece of window-glass.

[1] See page 94.

Directions. Hold the test-tube, half filled with water, above the flame, and warm the water, but do not let it boil. Above the mouth of the test-tube, and at a little distance from it, hold the glass plate, which should be cold, or the experiment may fail.

How did the particles of water which you see get on the piece of glass?

If the piece of glass had not been held over the mouth of the test-tube, what would have become of the water which now adheres to the glass?

If the test-tube were allowed to remain uncovered, what would become of the water in it after a time?

Definition. *Evaporation is the quiet formation of vapor at the surface of a liquid.*

Experiment 55. *To find what phenomena occur when water is heated gradually to boiling.*

Apparatus. A test-tube, provided with a small side tube, fitted with a perforated stopple with a glass tube thrust through, which reaches almost to the bottom of the test-tube; a bit of rubber tube to fit the side tube; a tumbler; a Bunsen burner; a piece of glass tube, bent at right angles, slipped, as shown in Fig. 38, into one end of the rubber tube; a clasp with which to attach the test-tube to the tumbler.

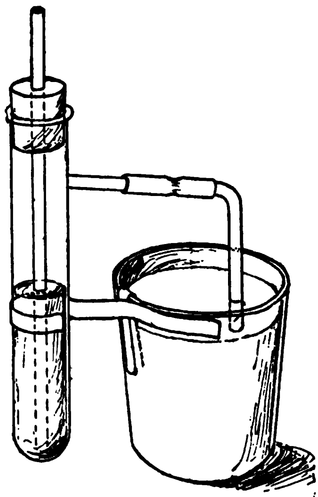


FIG. 38.

Directions. Draw some cold water from the tap, and fill the test-tube nearly to a level with the side tube. Put

the stopple with its tube in place in the mouth of the test-tube. Clasp the test-tube to the tumbler. Let the end of the glass tube, which you should attach to the side tube by means of the rubber tube, dip 1^{cm} or 2^{cm} beneath the cold water, which should nearly fill the tumbler. By applying heat to the bottom of the test-tube, gradually warm the water.

Before the water boils, bubbles appear in the test-tube.

Do these bubbles rise to the top of the water in the test-tube?

Are they bubbles of steam?

(It may be best to suspend judgment for a little while before answering this question.)

Do they burst at the surface?

Are they large or small?

As the water becomes warmer, more bubbles form.

In what part of the vessel do they form?

Do they rise to the surface of the water in the test-tube?

As they rise, do they grow larger or smaller?

Are they bubbles of steam?

(It may be best to suspend judgment for a little while before answering this question.)

In the upper part of the test-tube, is the water warmer or colder than at the bottom?

If there is a change of size in these bubbles as they rise, how do you account for it?

Do any bubbles appear at the end of the glass tube in the beaker of water? (When bubbles begin to appear in the beaker at the mouth of the tube, watch them carefully.)

As the water in the test-tube gets still warmer, do bubbles rise to the surface in either vessel?

How do you account for the sharp rattling noise (which is called *singing* or *simmering*) that is heard just before the water in the test-tube is brought to boiling?

After the bubbles have begun to come from the glass tube in the beaker, do they change in character as time goes on?

At the close of the experiment, do these bubbles detach themselves from the end of the tube?

After reflecting carefully on what you have observed, can you tell how to distinguish an air-bubble from a steam-bubble?

Is steam visible?

When water is boiling in a tea-kettle, what is the little cloud seen at the nose?

Keeping the test-tube in position and letting the end of the glass tube rest on the bottom of the beaker, remove the lamp and note what happens as the water begins to cool.

Do bubbles form?

Is the water in the test-tube boiling?

If the water is not boiling, what is the cause of what you observe?

Definition. *Boiling (or ebullition) is the rapid formation of vapor in the interior of a liquid.*

Experiment 56. *To find the temperature at which the aqueous vapor in the air will condense.*

Apparatus. A calorimeter, brightly polished; a thermometer; ice or snow; salt.

Directions. Into the calorimeter pour water drawn from the tap until it is of a depth sufficient to cover a little more than the bulb of the thermometer. By additions of ice-water or ice, with constant stirring, gradually cool this water until the brightly polished surface of the calorimeter becomes dimmed with dew. (Should no dew appear when the thermometer reads 0°, add salt and ice.) If at any time the water in the calorimeter becomes more than 5^{cm} deep, pour out the extra water. Record the temperature as soon as any dew is seen to appear on the calorimeter. Then allowing the temperature of the water to rise gradually by receiving heat from the room, record the temperature at which the dew begins to disappear.

Care should be taken not to let the breath come upon the cup, or near it; for the surface of the cup would become dimmed by the breath. By keeping some particular spot of the cup turned *from* you most of the time, and glancing at this spot frequently in order to detect the appearance or disappearance of dew upon it, you will more easily avoid errors from this source.

The average of the two temperatures recorded is taken as the dew-point.

Definition. *The temperature at which the air deposits some of its aqueous vapor is called the dew-point.*

The dew-point depends upon the amount of aqueous vapor in the air, and differs greatly from time to time. In general the dew-point is low in winter, but high in summer.

PECULIARITY IN THE EXPANSION OF WATER.

50. Maximum Density of Water. During the process of lowering the temperature of water to the freezing-point, the water contracts till it reaches a certain temperature, and then it begins to expand.

Experiment 57. *To find the temperature at which water is at its greatest density.*

Apparatus. A 250^{cc} flask; a rubber stopple, perforated with two holes, one to admit a tube 25^{cm} long of rather fine bore (2^{mm} or 3^{mm}), the other to admit a thermometer; a very narrow rubber band for a marker; ice or snow.

Directions. Avoiding air-bubbles, fill the bottle completely with cold water. Put in place the stopple, through which you have thrust the tube and the thermometer. The bulb of the thermometer should occupy a position midway between the bottom of the flask and its top. Slip the marker over the glass tube. After packing the flask in snow or broken ice as shown in Fig. 39, wait till the thermometer column becomes stationary. Bring the marker to a level with the top of the water column, which should also have become stationary. Then take the flask from the ice, and watch the motion of the water column as the temperature rises.

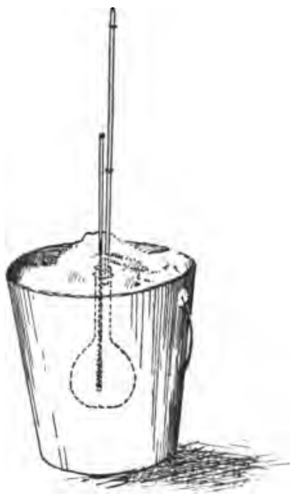


FIG. 39.

Before the column begins to rise, does it sink?

If the column sinks, what is the temperature of the water as indicated by the thermometer when the column is lowest?

From the action of the water column and the indications of the thermometer, what inference can you draw?

51. Results of the Peculiarity in the Expansion of Water brought out by the Last Experiment. In winter the surfaces of ponds and lakes lose heat. The water at the surface being, bulk for bulk, heavier than that below, sinks; this sinking of cold water goes on till the temperature of the whole pond is 4°. The surface water cools, but it is now less dense than the water below; this less dense water, therefore, floats, and its temperature falls till it freezes. The ice, as we know, floats. Deep bodies of water undisturbed by currents near the bottom or by springs, like Lake Tahoe in California, show a temperature at the bottom of about 4° all the year round. If water did not have this peculiarity in its expansion, the temperature of the whole pond would be reduced to 0°. Then, if the water still continued to contract on freezing, layer after layer of ice would form only to sink, until the whole pond would be a mass of solid ice.

In Art. 6 it was stated that *a cubic centimeter of water weighs 1^g;* this statement should now be amended to read *a cubic centimeter of water at 4° weighs 1^g.*

DISCUSSION OF A FEW TERMS.

52. Method. At the end of Chapter I. the meaning of observation and experiment, and the necessity of distinguishing between facts and inferences from the facts

were discussed. At this point you should turn back and review pages 62 and 63.

You have already learned that experiment is a most fruitful means of ascertaining facts.

The method, that is, the mode or way of accomplishing an end, differs for different experiments; but while experiments are infinite in number, the methods pursued in performing them are few, so it is possible, by marshalling the experiments each under its proper method, to bring order out of chaos. A most important method is known as the *method of differences*. Its utility depends upon varying only one condition at a time, while the precaution is taken to keep all the other conditions unchanged.

Interesting experiments according to the method of differences are not rare. "In his discovery of the cause of dew, Dr. Wells made use of this method. If on a clear calm night a sheet or other covering is stretched a foot or two above the earth, so as to screen the ground from the open sky, dew will be found on the grass around the screen but not beneath it. As the temperature and moistness of the air, and other circumstances, are exactly the same, the open sky must be an indispensable antecedent to dew. The same experiment is indeed tried for us by nature; for if we make observations of dew during two nights which differ in nothing but the absence of clouds in one and their presence in the other, we shall find that the clear sky is requisite to the formation of dew." [1]

53. Empirical Knowledge; Classified Knowledge. However useful *empirical knowledge*, the knowledge gained

[1] *Elementary Lessons in Logic*, by W. Stanley Jevons, p. 244.

by experience, may be, its importance is small as compared with *classified knowledge*, that is, knowledge well connected and perfectly explained. "He who knows exactly why a thing happens will also know exactly in what cases it will happen, and what differences in the circumstances will prevent the event from happening. Take, for instance, the simple effect of hot water in cracking glass. This is usually learned empirically. Most people have a confused idea that hot water has a natural and inevitable tendency to break glass, and that thin glass, being more fragile than other glass, will be more easily broken by hot water. Physical science, however, gives a very clear reason for the effect, by showing that it is only one case of the general tendency of heat to expand substances. The crack is caused by the successful effort of the heated glass to expand in spite of the colder glass with which it is connected. But then we shall see at once that the same will not be true of thin glass vessels; the heat will pass so quickly through that the glass will be nearly equally heated; and accordingly chemists habitually use thin uniform glass vessels to boil or hold hot liquids without fear of fractures which would be sure to take place in thick glass vessels or bottles."[1]

54. Cause; Hypothesis. By the *cause* of an event we mean the circumstances that must have preceded in order that the event should happen. An event has in general more than a single cause. "The cause of the boiling of water is not merely the application of heat up to a certain degree of temperature, but the possibility also

[1] *Elementary Lessons in Logic*, by W. Stanley Jevons, p. 257.

of the escape of the vapor when it acquires a certain pressure."[1]

Hypothesis means in science the imagining of a cause which underlies the phenomena we are examining, and is the agent in their production without being capable of direct observation. In making an hypothesis we assert the existence of a cause on the ground of the facts observed, and the probability of its existence depends upon the number of diverse facts it enables us to explain or reduce to harmony.

THE MOLECULAR HYPOTHESIS.

55. Molecular Hypothesis of Matter. The following brief account of the *molecular hypothesis of matter* in its modern form[2] may show more clearly how an hypothesis enables us to harmonize several different facts.

The material substances, as air, metals, wood, glass, water, and so on, are termed in the language of physics, *matter*.

All matter, according to the molecular hypothesis, is supposed to consist of exceedingly small particles, separated from each other by spaces. One of these exceedingly small particles is called a *molecule*.

"Every molecule consists of a definite quantity of matter, which is exactly the same for all molecules of the same

[1] *Elementary Lessons in Logic*, by W. Stanley Jevons, p. 239.

[2] The molecular hypothesis is a very old one. Lucretius, a Roman poet (95–55 B.C.), in his poem *De Rerum Natura* expounds the molecular hypothesis, and shows us that even in that remote period philosophers held the opinion that the observed properties of bodies apparently at rest are due to the action of invisible molecules in rapid motion.

substance. . . . The molecules of all bodies are in a constant state of agitation. The hotter a body is, the more violently are its molecules agitated. In *solid bodies*, a molecule, though in continual motion, never gets beyond a certain small distance from its original position in the body. The path which it describes is confined within a very small region of space.

"In *fluids*,[1] on the other hand, there is no such restriction to the excursions of a molecule. It is true that the molecule can generally travel but a very small distance before its path is disturbed by an encounter with some other molecule. . . . In fluids the path of the molecule is not confined within a limited region, as in the case of solids, but may penetrate to any part of the space occupied by the fluid."[2]

56. Evaporation and Condensation explained by the Molecular Hypothesis. The following explanation has been adapted from that given by Clerk Maxwell in his *Theory of Heat*. Some of the molecules of the liquid which are at the surface and are moving *from* the mass of the liquid may, by being struck by other molecules, get such velocities that they will escape from the forces which retain the other molecules in the liquid, and will fly about as vapor in the space outside the liquid. This is the way in which the molecular hypothesis explains evaporation. At the same time, a molecule of the vapor striking the liquid may become entangled among the molecules of the liquid, and may thus become part of the liquid. This is the

[1] The term *fluid* includes liquids and gases.
[2] *Theory of Heat*, bv J. Clerk Maxwell, p. 306.

explanation which the molecular hypothesis gives of condensation. The number of molecules that pass from the liquid to the vapor depends on the temperature of the liquid. The number of molecules that pass from the vapor to the liquid depends on the density of the vapor as well as on its temperature. If the temperature of the vapor is the same as that of the liquid, evaporation will take place as long as more molecules are evaporated than condensed; but when the density of the vapor has increased so much that as many molecules are condensed as evaporated, then the vapor has attained its maximum density. The vapor is then said to be *saturated*, and it is commonly supposed that evaporation ceases. According to the molecular hypothesis, however, evaporation is still going on as fast as ever; only condensation is also going on at an equal rate, since the proportions of liquid and gas remain unchanged.

EXAMPLES.

1. When the barometer is 77.8^{cm}, what is the error of the boiling-point of a thermometer that in freely escaping steam reads $100°.1$?

2. Explain why the temperature of boiling water depends upon the pressure.

3. Show the steps in the process of reasoning by which you found the coefficient of linear expansion of brass from the results of your experiment.

4. When the temperature is 0°, a copper lightning-rod measures 50 ft.; find its length in summer when heated to a temperature of 27°. The coefficient of linear expansion of copper is 0.0000173.

5. The coefficient of linear expansion of steel being 0.000012, what is the length at 0° of a bar that is just 1^{m} long at 20°?

6. A mass of air at 0° measures 200^{cc}; at 100° it measures 275^{cc}. Provided both measurements were taken under the same barometric pressure, find the coefficient of expansion of air.

7. When the temperature is 20°, a mass of oxygen gas measures 500^{cc}. When its temperature is raised to 40°, what will the volume of the gas become?

8. When the pressure is 38^{cm} and the temperature is 68°.25, a certain mass of air has a volume of 480^{cc}; what will this volume become when the pressure is 76^{cm} and the temperature is 0°?

9. Three liters of gas are measured off at 15° and 76.7^{cm} barometric pressure. Find the volume of this gas at the *standard temperature*, 0°, and at the *standard pressure*, 76^{cm}.

10. From the following data find the specific heat of nickel:

Weight of nickel	$= 400^g$.
" " water	$= 200^g$.
" " calorimeter	$= 100^g$.
Specific heat of calorimeter	= 0.1.
Temperature of nickel just before entering water	= 100°.
" " water " " nickel enters	= 15°.
" " the whole after " "	= 29°.7.

11. From the following data find the temperature after mixing water and mercury:

Weight of mercury	$= 1000^g$.
" " water	$= 100^g$.
Temperature of mercury	= 100°.
" " water	= 10°.
Specific heat of mercury	= 0.0333.
Number of units of heat absorbed by the calorimeter	= 80.

12. If the weight of a mass of liquid is 8^g, its temperature 80°, and its specific heat 0.24, how many grams of a second liquid, whose temperature is 10° and specific heat 0.48, must be mixed with the first in order that the resulting temperature may be 50°?

13. How many grams of a liquid whose temperature is 70° and whose specific heat is 0.25 must be poured on 20^g of ice at 0° in order to melt it? The latent heat of melting is 80.

14. How many grams of steam at 100° will be required just to melt 100^g of ice at 0°, if the latent heat of vaporization is 537, and the latent heat of melting is 80?

15. What temperature on the Centigrade scale corresponds to − 27° on the Fahrenheit?

CHAPTER III.

ELASTICITY.

57. Correction of the Reading of a Spring Balance when used in the Horizontal Position. In the present chapter the student will find a series of experiments on elasticity. Some knowledge of elasticity will prove helpful to him when he takes up the study of sound in the next chapter. As frequent use will be made of spring balances in our work, it will be well to consider the corrections that must be applied to their readings.

The maker of a spring balance intends to get the weight of the balance hook and the rod that connects the hook to the end of the spiral spring, which lies within the balance frame, so adjusted that, when the balance is hung up by its ring with no weight suspended from the hook, the pointer will be just opposite the zero line. Whenever a balance thus suspended has its pointer not opposite the zero line, the zero error is found by observing how far the pointer is above or below this line, as has already been mentioned on page 5.

In some of the subsequent experiments, however, we shall have occasion to use the spring balance held not in a vertical position, as was intended by the maker, but in a horizontal position. Whenever a spring balance with nothing hung on its hook is held in a horizontal position, the hook and the connecting rod no longer pull on the spring, so the spring contracts and draws the

pointer beyond the zero line until the pointer strikes against the end of the slot in which it moves, or, what amounts to the same thing, until a projection, near which the hook is fastened, in the connecting rod comes in contact with the balance frame. If we should lay the balance upon a table with its ring round an upright rod fastened in the table-top, and then pull in a horizontal direction on the hook, the number of pounds' or grams' pull, as registered by the pointer, would be too small by the weight of the hook and connecting rod; by adding, however, this weight to the amount registered by the pointer, we should get the number of pounds that we are really pulling on the hook. The purpose of the next experiment is to find this correction, that is, the weight of the hook and connecting rod.

FIG. 40.

Experiment 58. *To find the correction that must be applied to the reading of a given spring balance when the pull upon it is to be in a horizontal direction.*

Apparatus. Two 30-pound spring balances; two 8-ounce spring balances.

Directions. Suspend by its ring one of the 30-pound spring balances so that it shall hang freely; then hang by its hook the other 30-pound spring balance (see Fig. 40), the correction of which is to be found, to the hook of the spring balance already suspended. Record the reading of the upper spring balance. This reading is the weight of the lower spring balance. Record also the reading of the lower spring balance. This reading is the weight

of all of the lower spring balance except its hook and connecting rod.

What is the weight of the hook and connecting rod of the lower spring balance?

Repeat the experiment, using the pair of 8-ounce spring balances in place of the 30-pound spring balances.

In this case, what is the weight of the hook and connecting rod?

EXAMPLES.

1. From a spring balance which is suspended by its ring another spring balance is hung in an inverted position by its hook. If the spring balance suspended by its ring reads 3.7 oz., and the other spring balance reads 3 oz., what correction must be applied to the spring balance now reading 3 oz., when it is used in the horizontal position?

2. The spring balance, the correction for which was found in Example 1, is used in the horizontal position. The spring balance reads 5.3 oz., what is the true reading?

3. If the readings of two spring balances are the same as in Example 1, but the zero error of the spring balance which is being tested is 0.2 oz. (that is, the spring balance when suspended by its ring reads 0.2 oz. *above* the zero line), what would be the correction for the spring balance when used in the horizontal position?

Solution. If the spring balance reads 3.0 oz., its reading corrected for the zero error would be 3.2 oz., so the weight of its hook and rod would be $3.7 - 3.2 = 0.5$ oz. Hence 0.5 oz. must be added to the reading of the spring balance, when used in the horizontal position.

4. If -0.3 oz. is the zero error of a spring balance (that is, the spring balance when suspended by its ring reads 0.3 oz. *below* the zero line), and this spring balance reads 3.9 oz., when suspended by its hook from the hook of another spring balance which reads 4 oz., what correction must be applied when the balance is used in the horizontal position?

5. A spring balance the zero error of which is 0.4 lb. is suspended by its ring, and from its hook is hung in an inverted position another spring balance the zero error of which is -0.3 lb. If the reading of the spring balance suspended by its ring is 3.2 lbs., and that of the other 3.0 lbs., what correction must be applied to the reading of the spring balance which has been thus tested, when used in the horizontal position?

FORCE.

58. Meaning of the Term Force. The term *force* is important in physics, and it will be our object in the earlier of the following experiments to make the meaning of this term clear.

Experiment 59. *To find the tension necessary to break a wire.*

Apparatus. A 30-pound spring balance; 5 pieces of No. 30 B. & S. spring brass wire, each about 1^{m} long; a wooden guard to slip on hook of the balance.

NOTE. No. 30 B. & S. means that the size of the wire is number 30, measured on the Brown and Sharpe wire-gauge.

Directions. Round some vertical cylindrical object like a gas-pipe make several turns with one end of one of the pieces of wire, and to prevent slipping fasten this end to a tack driven into the woodwork to which the pipe is fastened. Pass the other end through the eye where the hook is attached to the balance, and fasten by twisting the wire about itself. Over the hook slip the wooden guard, and wind the wire round it several times, as shown in Fig. 41, taking care there is no slack wire between the hook and the guard. See that there are no kinks in the wire. Holding the balance in a horizontal position, and taking care that the rod of the balance to which the hook is attached does not bind or rub against the frame of the balance, gradually increase the tension, and constantly watch the pointer until the wire breaks. When holding

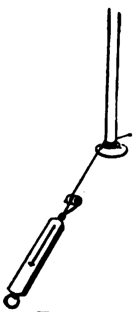


FIG. 41.

the balance, avoid having the hand near the end to which the hook is attached, lest the finger be injured by the recoil.

Record the error of the balance when used in the horizontal position. Also record the number of pounds the balance indicates when the wire breaks.

Why were you directed to keep the wire free from kinks, and to wrap it round cylindrical objects like the gas-pipe and the wooden guard?

Using a new piece of wire each time, make in all five tests. Record the result of each trial, and find the average, corrected for the error of the balance.

The tension, or pull, required to break the wire is called the *breaking strength* of the wire.

Find the product of 453.6, and the *number* that expresses the average breaking strength. This product will be the number that expresses the average breaking strength in grams, as in 1 lb. there are 453.6^g.

Experiment 60. *To find how long a wire must be to break under its own weight, if suspended by one end.*

Apparatus. A horn-pan balance; 1^m of No. 30 B. & S. spring brass wire; a micrometer gauge.

Directions. Measure off accurately 1^m of the wire. Weigh it to 0.1^g on the horn-pan balance.

If the 1^m of wire were suspended by one end, how hard would it pull down at the point of support?

Turn back to the last experiment, and get the record of the breaking strength of the wire in grams.

Knowing the weight of 1^m of the wire and its breaking strength, find by computation the length of a piece that would break of its own weight, if supported at one end.

With the micrometer gauge measure the diameter of the wire; and find area of cross-section of wire, using formula $A = \frac{1}{4}\pi D^2$. Knowing that the strength of wires of the same material as well as their weight, for equal lengths, is proportional to their area of cross-section, find by computation the breaking strength of a brass wire $1^{sq\,cm}$ in area of cross-section.

ELASTICITY OF STRETCHING.

Experiment 61. *To find whether a wire will increase by equal amounts in length for equal additions to the tension by which it is stretched.*

Apparatus. About 4^m of No. 27 B. & S. spring brass wire; a 30-pound spring balance; two meter sticks graduated in millimeters; a wooden screw and nut; an iron screw; two narrow mirrors.

NOTE. The wooden screw and nut can be readily obtained from a wooden clamp such as carpenters use. An inspection of Fig. 42 will show how the screw and nut are arranged. The side of the clamp next the handle of the screw has a slot cut in it to fit the edge of the table. The nut is made by sawing off a piece of the side containing the thread in which the screw works, and should have a strong hook fastened to it.

Directions. Into the top of a table, near one end, insert the iron screw; and solder one end of the wire to it. By passing the other end of the wire through the eye in which the balance hook is fastened, and twisting this end round the wire, make it fast. Wind the wire round the hook of the balance several times.

Put the wooden screw and nut at the end of the table remote from the iron screw. Put the ring of the balance over the hook on the nut, and, by turning the screw, make the balance indicate a tension of 1 lb. This tension will

draw the wire straight. Now solder[1] two pieces of fine wire, each about 0.5^{cm} in length, across the long wire at right angles, one near each end, to serve as markers. As shown in Fig. 42, lay a meter stick lengthwise under each end of the wire. Lay on each of the meter sticks a narrow mirror, and adjust each meter stick, by sliding it back and forth, till the marker under which it lies is exactly

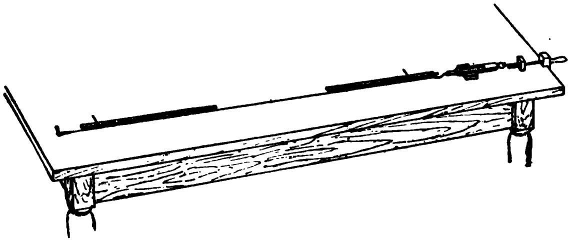


FIG. 42.

over one of the divisions of the meter stick, when the eye is held, as it should always be held in making readings in this experiment, so that the marker obscures its own image. Do not allow the meter sticks to be moved during the experiment.

By turning the screw increase the tension by 2 lbs. (The balance now reads 3 lbs.) Read the position of each

[1] In order to solder two pieces of brass, brighten the parts to be joined by careful rubbing with a file or with emery cloth. Moisten these bright parts with soldering-fluid, and over each of them pass the tip of a hot soldering-iron on which is a drop of melted solder to "tin" or coat the surfaces. Lay the tinned surfaces together, moisten them with soldering-fluid, and press upon them the hot soldering-iron, from the tip of which a drop of solder is allowed to flow and to spread round and between the surfaces. Remove the soldering-iron, and let the solder harden at the juncture. Do not heat the soldering-iron after a greenish flame begins to play round it, lest the tinning at its tip be destroyed.

marker to quarters of a millimeter; the difference between their movements is the amount the portion of the wire lying between them has stretched. Reduce the tension to 1 lb., and notice whether the wire regains its original length.

Make several trials similar to this, increasing the force 2 lbs. at a time (making the balance read in addition to the 1 lb., which is always kept applied, 4 lbs., and then 6 lbs.) until the wire fails to return to its original length, when the tension acting on the wire is reduced to 1 lb.

In recording the results, do not count the force of 1 lb. applied to keep the wire straight.

The results may be recorded as follows:

MATERIAL OF WIRE .... GAUGE NO. .... LENGTH ....

TENSION.	MOVEMENT OF FIRST MARKER.	MOVEMENT OF SECOND MARKER.	ELONGATION.	ELONGATION PER POUND.
lb.	mm.	mm.	mm.	mm.
....	....	....	....	....
....	....	....	....	....
....	....	....	....	....

Measure and record in millimeters the length of the wire between the two markers.

For what value of the tension does the wire fail to return to its original length, after the pull on the wire has been reduced to 1 lb.?

Does the amount of elongation show any simple relation to the amount of tension?

(HINT. In answering this question, look at the first column and at the fourth of your record.)

The answer to the last question, which should be modelled after the statement of Laws 2 and 3 below, constitutes what we shall call Law 1 for the stretching of a wire.

Law 2. For a given tension, elongation is proportional to the length of the wire; that is, the longer the wire, the greater the elongation.

Law 3. For a given tension, the elongation is *inversely* proportional to the area of cross-section of the wire; that is, the larger the area of the cross-section of the wire, the less the elongation.

How could you prove experimentally Law 2? Law 3?

In Exp. 59 the tension was so great that the wire broke, while in the present experiment the tension simply stretched the wire. A *tension* or *pull* is called a *force*. In Exp. 28 the air confined in the closed branch of the Boyle's tube was compressed or pushed into a smaller volume by the weight of the mercury. This *compressing action* or *push* is also called a *force*.

Definition. *A force is a push or a pull.*

The power which a body has of recovering, more or less completely, its original shape after the force which has changed the shape is withdrawn, is called *elasticity of shape* or *figure*.

If stretched or compressed within certain small limits (that is, stretched or compressed only a little), most solid bodies will return to their original dimensions, after the forces, to which they have been exposed, cease to act. These limits are called *the limits of elasticity*.

In this experiment, did you stretch the wire beyond its limits of elasticity?

ELASTICITY OF BENDING.

59. Effects of Bending. In the preceding experiment we observed that a wire had the power of recovering its original length more or less completely when the force which stretched it ceased to act. In the next four experiments we shall continue our work in elasticity by studying the effects produced by bending rods of different dimensions.

Experiment 62. *To find the relation between the load and the amount of bending produced in a rod supported at each end.*

Apparatus. A straight rod of clear white pine a little more than 100^{cm} long, and about 1^{cm} wide, and 1^{cm} thick; three triangular prisms of wood; weights from 100^{g} to 400^{g}; a scale about 10^{cm} long with a support to keep it in a vertical position; a very light, thin rod of wood about 32^{cm} long, called a pointer; a meter stick.

Directions. Lay the rod in a horizontal position on two of the triangular blocks, placed parallel to each other with their centers 1^{m} apart. The rod should be parallel to the edge of the table and a little more than 30^{cm} from the edge. Place the remaining block opposite the middle of the rod and parallel to it. The center of this block is to be 5^{cm} from the nearer edge of the rod, as shown in Fig. 43. By placing one end under the center of the rod, support the pointer on the block, so that it lies across the block, and at right angles to the rod. Place with its back turned towards the rod, in a vertical position, the 10^{cm} scale at a distance of 30^{cm} from the edge of the rod, and close beside the pointer. The movement of the end of

the pointer will magnify the bending of the rod five times. The support on which the pointer rests should be a little higher than the supports of the rod, so that the pointer will rest against the front edge of the rod throughout the experiment, and so continue to magnify the bending of the rod five times.

Sight carefully across the upper surface of the pointer, and record its position on the 10^{cm} scale. Then on the middle of the rod lay carefully a 100^{g} weight, and again

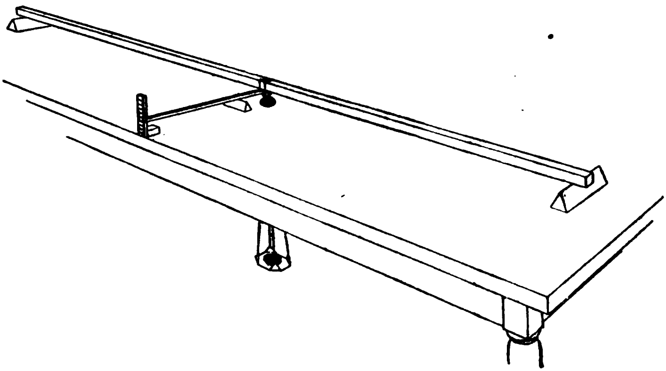


FIG. 43.

read the position of the pointer. (If the weights are not flat, lay the weights in a little pan, as shown in Fig. 43.) Remove this weight, and read and record again the position of the pointer. The weight applied to the rod is called the *load*. Now put on 200^{g}, and read and record the position of the pointer. Also take the reading of the pointer with the load off. Then add 300^{g} and 400^{g} in turn, reading and recording the position of the pointer every time the load is put on, and also when it is taken off.

If permanent bending should be observed, stop recording the readings.

When weights are put on and taken off, see that the rod is not shifted on its supports.

The following form for recording observations is suggested :

SUPPORTS 100^{cm} APART. WIDTH OF ROD .... THICKNESS OF ROD ....

LOAD.	READING WITH LOAD.	READING WITHOUT LOAD.	AVERAGE READING WITHOUT LOAD.	RISE OF POINTER ON SCALE.	DEFLECTION OF ROD.	DEFLECTION PER 100^{g}.
g.	cm.	cm.	cm.	cm.	cm.	cm.
		3.00				
100	3.40		3.00	0.40 ÷ 5	= 0.080	0.080
		3.00				
200	3.75		2.98	0.77 ÷ 5	= 0.154	0.077
		2.96				

Find the average of the numbers in the column headed "Deflection per 100^{g}."

Divide each load in the first column of your observations by the first load (100^{g}); also divide each deflection in the sixth column by the first deflection (the deflection produced by a load of 100^{g}).

From the results thus obtained, what should you say is the relation between the load and the bending produced?

QUESTION. If a load of 100^{g} produces a bending of 0.25^{cm}, what will be the amount of bending produced by a load of 300^{g} ?

NOTE. Sometimes the load is referred to as the *transverse force*. The expressions *amount of bending* and *amount of flexure* have the same meaning, signifying the distance through which the middle of the rod is depressed.

Experiment 63. *To find the relation between the length of rods and the amount of bending produced by equal loads.*

Apparatus. The same as in the last experiment; weights from 500^g to 2000^g.

Directions. Arrange the apparatus in all respects as in the last experiment, with the exception of having the supports placed at a distance of 50^{cm} instead of 100^{cm} apart. Let the middle portion of the rod be subjected to the bending, that is, let about 25^{cm} of each end of the rod project over each support.

Using the same precautions as already mentioned for the preceding experiment, and adding 500^g at a time, load from 500^g to 2000^g. Record as before, and find the average of the deflections per 100^g.

Divide the average of the deflections per 100^g of the last experiment by the corresponding average of the present experiment; also divide the length (the distance between the supports) of the longer rod by that of the shorter.

If the quotients thus obtained are not equal, try to make them equal by squaring or cubing one of them.

What relation should you say exists between the length of rods and their amount of flexure?

QUESTION. If a rod 100^{cm} long, when loaded with 100^g, is bent 0.12^{cm}, how much will the same load bend a rod 50^{cm} long?

Experiment 64. *To find the relation between the breadth of rods and the amount of bending produced by equal loads.*

Apparatus. With the exception of the rod and weights, the same as in the last experiment; a straight rod of clear white pine a little more than 100^{cm} long, and about 1^{cm} thick and 2^{cm} wide; weights from 200^g to 800^g.

Directions. Lay the rod on its broad side with the supports 100^{cm} apart. Adding 200^{g} at a time, load from 200^{g} to 800^{g}. Record as before.

In this experiment and in Exp. 62 we have rods alike in all respects except that of width. Divide the average deflection per 100^{g} of the rod of Exp. 62 by the corresponding average for the rod of this experiment; also divide the width of the rod of the present experiment by the width of the rod used in Exp. 62.

Are the two quotients equal or nearly equal?

What relation can you make out as probably existing between the breadth of rods and the amount of flexure for equal loads?

QUESTION. If a rod 1^{cm} wide is bent 0.12^{cm} by a load of 100^{g}, how much will the same load bend a rod, of equal length and thickness, and 2^{cm} wide?

Experiment 65. *To find the relation between the thickness (depth) of rods and the amount of bending produced by equal loads.*

Apparatus. With the exception of the weights, the same as in the preceding experiment; weights from 500^{g} to 2000^{g}.

Directions. Use the broad rod of the preceding experiment. Place this rod on edge with the supports 100^{cm} apart. Adding 500^{g} at a time, load from 500^{g} to 2000^{g}. Record as before.

This rod is of the same dimensions as that used in Exp. 62, with the exception of that of thickness (depth). Divide the average deflection per 100^{g} of the rod of Exp. 62 by the corresponding average of the rod of this experiment; also divide the thickness of the rod of this experiment by that of the rod used in Exp. 62.

If the two quotients are not equal, try to make them equal by squaring or cubing one of them.

What relation can you make out as probably existing between the thickness of rods and the amount of bending for equal loads?

In the experiments on bending, have you tested the *strength* or the *stiffness* of the rods?

QUESTION. If a load of 100^{g} bends a rod 1^{cm} thick 0.12^{cm}, how much will the same load bend a rod 2^{cm} thick?

ELASTICITY OF TORSION.

60. Effects of Twisting. The *elasticity* of *torsion*, or *twisting*, is shown by the alternate twisting and untwisting when a weight is suspended from an ordinary string.

Experiment 66. *To find the relation between the amount of twisting of a rod and the force applied.*

Apparatus. A rod of clear ash about 1^{m} long and ½ by ½ in. in cross-section; one end of this rod is fitted into the middle of a circular board 1 ft. in diameter; a sheet of cardboard upon which is traced a circle whose diameter is somewhat greater than that of the circular board, and whose circumference is divided into degrees; two 8-ounce spring balances; a narrow mirror to be used as in Exp. 61.

Directions. Upon a table, having uprights at each end which support a movable cross-piece, place the cardboard with the graduated circle uppermost, with the hole in its center over the hole in a metallic plate set into the table-top under the cross-piece. Place the rod upright with the stout pin, which projects from the center of the board, in the hole in the table-top. Fasten by a clamp, as shown

in Fig. 44, the other end of the rod in the long slot in the movable cross-piece above the table. The length of the rod from the top of the circular board to the cross-piece must be 80^{cm}. To get the right length, adjust the cross-piece. Do not let the circular board rub on the cardboard;

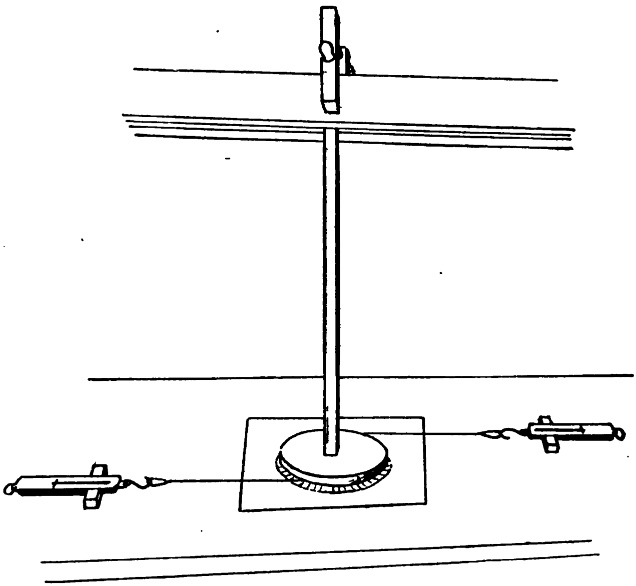


FIG. 44.

do not let the peg bind in the hole. By means of tacks, fasten the cardboard at the corners to the table. The circle on the cardboard and circular board must be concentric. Have the zero mark on the circle just beneath a pin driven into the edge of the circular board for an index.

To each of the screws inserted, from diametrically opposite points, into the edge of the circular board, attach a

piece of flexible string. To each string attach an 8-ounce spring balance, and with these balances pull horizontally in opposite directions at right angles to the line joining the screws. Let the force be the same for each balance, and, as the rod twists, let the direction of the pull be changed in such a way as to keep it always at right angles to the line joining the screws. Use in succession forces of 2, 4, 6, and 8 oz., and record in each case the resulting torsion (that is, the number of degrees) as shown by the readings on the cardboard scale. Of course, as shown in Exp. 58, a correction must be added to the indication of the spring balance, when used in the horizontal position, to get the true measure of the force; so this correction for each spring balance must be found and recorded.

The following form of record is suggested:

ROD OF CROSS-SECTION ½ BY ½ IN.; LENGTH 80cm.

TWISTING FORCE.	READING WITH TWISTING FORCE.	READING WITHOUT TWISTING FORCE.	AVERAGE READINGS WITHOUT TWISTING FORCE.	TWIST OF ROD.	TWIST OF ROD PER 1 OZ.
oz.	°	°	°	°	°
		1			
2	3		1	2	1
		1			
4	5		1	4	1
		1			

Divide each twisting force in the first column of your record by the first twisting force (2 oz.); also divide each twist of rod in the fifth column by the first twist (produced by the twisting force of 2 oz.).

From the results thus obtained, what should you say is the relation between the force applied and the twisting produced?

QUESTION. If a force of 5 oz. twists a certain rod 1.5°, through how many degrees will a force of 12 oz. twist the rod?

Experiment 67. *To find the relation between the amount of twisting for equal forces and the lengths of rods.*

Apparatus. The same as in the preceding experiment, but instead of the two 8-ounce spring balances, two 4-pound spring balances.

Directions. Arrange everything as in the preceding experiment, but lower the cross-piece so that only 40^{cm} of the length of the rod shall be subject to torsion. Find and record the corrections for the spring balances, and, using forces of 4, 8, 16, and 32 oz., proceed as before and record the results.

Turn back to Exp. 66 to find the average twist of rod per ounce. Divide this twist by the corresponding average twist of the rod of the present experiment. Divide the length of the rod of Exp. 66 by that of the rod of the present experiment.

Are the quotients equal or nearly equal?

Can you state a probable relation between the twisting for equal forces and the length of rods?

QUESTION. If a force of 8 oz. twists a rod 100^{cm} long 3°, through how many degrees will the same force twist a rod whose length is 50^{cm}?

Experiment 68. *To find the relation between the diameters of the cross-section of rods and the amount of twisting produced by equal forces.*

Apparatus. The same as in the preceding experiment, except that a rod of clear ash of cross-section $\frac{3}{4}$ by $\frac{3}{4}$ in. is used.

Directions. Clamp the rod so that a length of 80^{cm} shall be subject to torsion. Find and record the corrections for the spring balances. Apply forces of 1, 2, 3, and 4 lbs. (that is, 16, 32, 48, and 64 oz.). Record as before.

Divide the average twist per ounce of the rod (whose diameter is ½ in.) of Exp. 66 by the average twist of the rod (whose diameter is ¾ in.) of the present experiment; also divide the diameter of the rod of the present experiment by that of Exp. 66.

How nearly do your results agree with the statement, "The amount of torsion is inversely proportional to the fourth power of the diameter"?

Question. If a rod whose diameter is 1 in. is twisted by a certain force through an angle of 5°, through how many degrees would the same force twist a similar rod whose diameter is 2 in.?

61. Elasticity of Volume. Besides *elasticity of shape* or *figure*, a body possesses *elasticity of volume*, or the power of recovering, more or less perfectly, its original volume after the force which has changed the volume is withdrawn.

Elasticity of volume is possessed in perfection by liquids and gases, which recover completely their original volumes when the compressing forces are removed, no matter how long they have been applied. In Exp. 28 the weight of mercury in the open branch compressed the air in the closed branch, thus making it occupy a smaller volume; but when the pressure is removed, the air, by reason of its elasticity of volume, returns to its former bulk.

62. Strain; Stress; Hooke's Law. Any change in the shape or size of a body is called a *strain*. Any application of force tending to produce a strain is called a *stress*.

In Exp. 61 the elongation of the wire is a strain, and the tension that produced the elongation is a stress.

In the experiments on bending and twisting, the student should point out the strain and the stress.

NOTE. Many years ago an Englishman, named Hooke, discovered Law I. of stretching; hence this law is often called Hooke's Law. Hooke stated his law in the following words: "*Ut tensio sic vis.*"

Using the words *stress* and *strain* as just defined, could Hooke's Law be stated thus?

"*The stress is proportional to the strain.*"

EXAMPLES.

Before working the following set of examples, collect into a group the three laws of stretching (see Exp. 61), the four laws of bending (see Exps. 62, 63, 64, and 65), and the three laws of torsion (see Exps. 66, 67, and 68).

1. A balance, the zero error of which is −0.2 lb., weighs 1.7 lb. When this balance is suspended by its hook, the pointer indicates 1.5 lb. What correetion must be applied to the reading of the balance when it is used in the horizontal position?

2. If a wire 3^{m} long and $0.2^{sq\,mm}$ in area of cross-section is stretched 2.5^{mm} by a force of 2^{kg} (2000^{g}), how great a force would be required to stretch by 3^{mm} a wire of like material 12^{m} long and $2^{sq\,mm}$ in area of cross-section?

Solution. If a wire 3^{m} long and $0.2^{sq\,mm}$ in area of cross-section is stretched 2.5^{mm} by a force of 2^{kg}, a force of 1^{kg} would stretch the wire one-half as much, or $\frac{2.5}{2}^{mm}$ (stretching is proportional to the tension), and this force of 1^{kg} would stretch a length of 1^{m} of the wire one-third as much, or $\frac{2.5}{6}^{mm}$ (stretching is proportional to the length). Finally, if the area of cross-section of the wire were $1^{sq\,mm}$, a length of 1^{m} of the wire would be stretched by a force of 1^{kg}, two-tenths as much, or $\frac{0.5}{6} = \frac{1}{12}^{mm}$ (stretching is inversely proportional to the area of the cross-section).

If by x we denote the force required to stretch the second wire 3^{mm}, we shall find by a process precisely like the one just given that a wire 1^{m} long and $1^{sq\,mm}$ in area of cross-section will be stretched $\frac{1}{2x}^{mm}$ by a force of 1^{kg}.

As the material is the same for each wire, we have

$$\frac{1}{2x} = \frac{1}{12}$$
$$2x = 12$$
$$\therefore x = 6$$

Hence, the force required to stretch the second wire will be 6^{kg}.

3. If a wire 8^{m} long and $0.4^{sq\,mm}$ in area of cross-section is stretched 10^{mm} by a force of 5^{kg}, how great a force would be required to stretch by 5^{mm} a wire of like material 20^{m} long and $4^{sq\,mm}$ in area of cross-section?

4. If a wire 10^{m} long and 1^{mm} in diameter is stretched 10^{mm} by a force of 10^{kg}, how many millimeters would a force of 8^{kg} stretch a wire of like material 25^{m} long and 2^{mm} in diameter?

5. What is the ratio of the stiffness of a rod 60^{cm} long to that of another rod 120^{cm} long, but of the same width, thickness, and material as the first?

6. If a rod 4 ft. long, 2 in. broad, and 0.5 in. thick is bent 0.1 in. by a weight of 10 lbs., how much would a force of 2 lbs. bend a rod of like material 12 ft. long, 4 in. broad, and 2 in. thick?

Solution. It a rod 4 ft. long, 2 in. broad, and 0.5 in. thick is bent 0.1 by a weight of 10 lbs., a weight of 1 lb. would bend it one-tenth as much, or $\frac{0.1}{10}$ in. (bending is proportional to the load), and this weight of 1 lb. would bend a rod 1 ft. long only one sixty-fourth as much, or $\frac{0.1}{640}$ in. (bending is proportional to the cube of the length). If the rod were 1 ft. long and 1 in. wide, the weight of 1 lb. would bend it twice as much, or $\frac{0.2}{640}$ in. (bending is inversely proportional to the breadth). Finally, if the thickness of the rod were 1 in., its length 1 ft., and its breadth 1 in., a force of 1 lb. would bend it one hundred and twenty-five thousandths as much, or $\frac{0.025}{640} = \frac{0.1}{2560}$ in. (bending is inversely proportional to the cube of the thickness).

If by x we denote the amount the second rod would be bent by a weight of 2 lbs., we shall find by a process precisely like the one just given that a rod 1 ft. long, 1 in. broad, and 1 in. thick will be bent $\frac{x}{108}$ in. by a force of 1 lb.

As the material is the same for each rod, we have

$$\frac{x}{108} = \frac{0.1}{2560}$$

$$2560x = 10.8$$

$$\therefore x = 0.0042$$

Hence, the amount the second rod would be bent is 0.0042 in.

7. If a rod 4^{m} long, 6^{cm} wide, and 8^{cm} thick is depressed 0.75^{cm} at its middle point by a certain load, how much would the same load depress a rod 3^{m} long, 3^{cm} wide, and 4^{cm} thick?

8. If a rod 100^{cm} long, 2^{cm} broad, and 3^{cm} thick is deflected 0.5^{cm}, what would be the deflection, under the same load, of a rod 50^{cm} long, 2^{cm} broad, and 1^{cm} thick?

9. If a certain beam 16 ft. long, 4 in. wide, and 6 in. thick is bent 1 in. by a load of 500 lbs. placed at its middle, how much would a beam 10 ft. long, 8 in. wide, and 12 in. thick be bent by the same load?

10. A certain beam 4 ft. long is bent downwards 0.5 in. by a load placed at the middle. If it were 8 ft. long, how far would it be bent by the same load?

11. There are two beams of the same length, breadth, and material. One beam, which is 8 in. thick, is bent 1.6 in. by a certain load, while the other beam is bent 0.2 in. by an equal load. What is the thickness of the second beam?

12. If a rod 80^{cm} long is twisted through an angle of 1.5° by a force of 4 oz., through how many degrees will a force of 3 oz. twist a rod 100^{cm} long, the other dimensions as well as the material being the same as that of the first rod?

CHAPTER IV.

SOUND.

63. Wave-Motion. If a stone is dropped into a pool of calm water, the stone immediately forces down and displaces a number of particles of water; consequently the surrounding particles of water are heaped above the general level; these descend and throw up another wave, and this in subsiding raises another, until the force of the original and loftier wave dies away at the edge of the pool in the faintest ripples. You have probably noticed the waves that spread in ever-widening circles over a pool when the water has been disturbed as described. Did you ever ask yourself the question, "Do the particles of water forming the first wave, in the center of the pool, pass to the second wave, and so on to the third, and finally reach the margin of the pool?" This question is easily answered by watching the movements of a bit of wood floating on the surface. The wood simply bobs up and down; it does not approach the shore. Since the wood only rises and falls, we see that the particles of water on which it rests are not approaching the shore, but are only moving up and down. Hence it is not the *particles of water*, but the *wave form* that travels from the center to the margin of the pool. When in summer a wind blows over a field of grain, we see the wave form advancing, but we know that the ears of grain are simply nodding.

If into a pool of water we drop two pebbles at a little distance apart, we shall notice that two sets of waves are produced, each set having for its center the point where the pebble entered the water. As the waves spread, those of one set cross those of the other. If we look carefully, we shall see that where the *crest*, or top, of one wave coincides with the crest of another, an elevation higher than the crest of either wave is produced; also, that where the *trough*, or hollow, of one wave coincides with the trough of another, a depression deeper than the trough of either is formed; finally, if two waves of equal size come together in such a way that the crest of one coincides with the trough of the other, there is neither an elevation nor a depression, but a *calm*.

The distance from crest to crest, or from trough to trough, is called a *wave-length*.

Definition. *A wave-length is the distance from any particle to the next particle that is in a similar position in its path, and is moving in the same direction.*

In Fig. 45 take the particle *a*; *b* is in the same position, but when *a* is moving downwards *b* moves upwards.

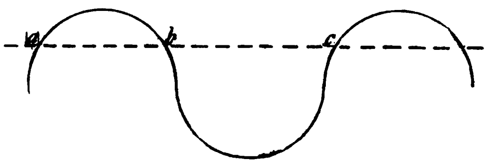


FIG. 45.

c is the next particle in the same position, but as *a* moves downwards *c* moves downwards. The distance from *a* to *c* measured in a straight line is a wave-length.

Let the student, by dipping his finger into the water contained in a pail and then quickly removing it, set up a series of waves as has already been described.

After the waves strike the sides of the pail, are they sent back towards the center?

By using a finger of each hand, disturb the water in such a way as to produce two series of waves. One series mingles with the other.

Do you notice any phenomena of interference (that is, calms and places of greater or less elevation produced by the mingling of the waves)?

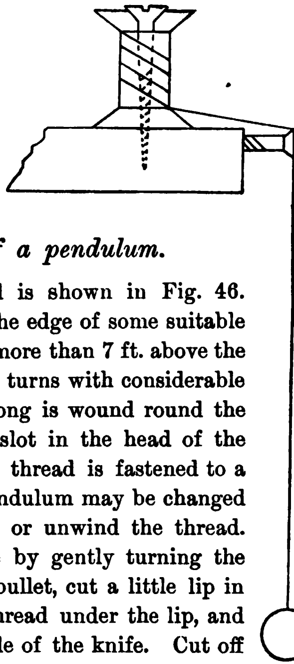
You should repeat these observations till the eye becomes trained to catch the modifications of the wave form.

THE PENDULUM.

64. The Simple Pendulum. When the surface of water is agitated, the particles of water move up and down with a rythmic motion. It is important to understand the chief laws this rythmic motion obeys. The motion of the pendulum is of the same nature as that of the water particles, so by experimenting with the pendulum, we shall get some knowledge of the motion of the water particles.

The pendulum we shall use will consist of a lead bullet attached to a firm support by a thread, which will allow the bullet to swing freely. The bullet is called the *bob* of the pendulum. The fixed point to which the thread is attached is called the *center of suspension* of the pendulum. A motion from side to side is called an *oscillation;* a motion *from one side to the other and back again* is called a *vibra-*

tion, so that there are two oscillations in one vibration. The distance the bob swings through, in going from its middle position to its extreme position, is called the *amplitude* of vibration. The distance from the *center of suspension* to the *center of the bob* is called the *length of the pendulum*. The length of time a pendulum takes to make an oscillation is called the *time of oscillation*.



Experiment 69. *To find whether a change of amplitude has any effect upon the number of vibrations per minute of a pendulum.*

Apparatus. The apparatus used is shown in Fig. 46. A spool is fastened by a screw near the edge of some suitable support, fastened to the wall a little more than 7 ft. above the floor. The screw is "set up" till it turns with considerable friction. A silk thread about 3^m long is wound round the spool and made to pass through the slot in the head of the other screw. The lower end of the thread is fastened to a bullet, the bob. The length of the pendulum may be changed by turning the spool so as to wind or unwind the thread. Small adjustments are easily made by gently turning the spool. To fasten the thread to the bullet, cut a little lip in the bullet with a knife, place the thread under the lip, and smooth the lip down with the handle of the knife. Cut off the short end of the thread close to the bullet. A watch having a second hand is also necessary.

FIG. 46.

Directions. Taking the lower end of the slot of the screw as the center of suspension, make the pendulum 36^{cm} long. Fill the slot with beeswax so that the thread will not move in the slot. Draw the bob about 5^{cm} to

one side and release it. The pendulum should swing parallel to the wall to which the support of the pendulum is fastened. Glance at the watch and at the pendulum. When the pendulum is just starting to return from the end of its swing, take the time. Count accurately for one minute the number of vibrations. Record the number of vibrations. Double the amplitude of vibration (by drawing the bob at the start about 10^{cm} to one side) and count the number of vibrations for another minute.

What effect has a change of amplitude on the number of vibrations performed by a pendulum in a minute?

Experiment 70. *To find the effect a change of length in a pendulum has upon the number of vibrations performed in a minute.*

Apparatus. The same as in the last experiment; a long pole.

Directions. Place one end of the long pole on the floor, so that the pole shall stand in a vertical position with its side against the screw. Mark on the pole the position of the center of suspension; then, starting from this point, measure off on the pole a distance of 225^{cm}. Put the pole in the vertical position as before and adjust the bob till the pendulum is 225^{cm} long.

Using one amplitude, record, as before, the number of vibrations for one minute. Then make the length of the pendulum 100^{cm}, and record the number of vibrations for one minute. Finally make the pendulum 25^{cm} long, and record as before.

Divide the greatest length of the pendulum by the least. Divide the largest number of vibrations by the smallest.

Is there any agreement between the quotients?

Does taking the square root of one of the quotients make the values approximately equal?

What is the relation between the lengths of pendulums and the number of vibrations?

QUESTIONS. From an inspection of your results, answer the following questions: What is the length of a pendulum that will beat seconds? half seconds?

Experiment 71. *To find whether the weight and material of the pendulum bob have any effect on the number of vibrations in a minute.*

Apparatus. The same as in Exp. 69; a marble of about the same size as the bullet.

Directions. In place of the bullet, fasten the marble to the end of the silk thread by means of beeswax. Make the pendulum 36^{cm} long. Record the number of vibrations per minute.

Compare the result with that recorded in Exp. 69.

What is your conclusion?

EXAMPLES.

1. If a pendulum makes 30 vibrations in a minute, how many oscillations does it make in 2 minutes? How many oscillations does the pendulum make in a second?

2. If a pendulum makes 40 vibrations in a minute, what is the time of a single vibration?

3. If a pendulum 25^{cm} long makes 60 vibrations in a minute, what must be the length of a pendulum in order that it shall make 10 vibrations in a minute?

Solution. As the number of vibrations of two pendulums are to each other inversely as the square roots of the lengths of the pendulums, we have, if we denote by x the required length,

$$60 : 10 = \sqrt{x} : \sqrt{25}$$
$$10\sqrt{x} = 60\sqrt{25}$$
$$\sqrt{x} = 6\sqrt{25}$$
$$\therefore x = 900$$

Hence the required length is 900^{cm}.

4. If a pendulum 1^m long vibrates once in a second, what must be the length of a pendulum in order that it shall oscillate once in a minute?

5. If a pendulum 4 units in length makes a vibration in 0.3 second, find the length of a pendulum that makes a vibration in 1.8 seconds.

6. If a pendulum 6 units in length makes an oscillation in 5 seconds, what will be the time of an oscillation of a pendulum 0.54 units in length?

VELOCITY OF SOUND.

65. The Velocity of Sound in Air. Our next experiment will be for the purpose of getting, roughly, the velocity of sound in air; of getting, in other words, the distance that sound will travel in one second.

Experiment 72. *To find the velocity of sound in air.*

Apparatus. A wooden support like the one used in Exp. 28; a bullet; a piece of silk thread; a stool; two small boards; a small spy-glass.

Directions. This experiment is to be performed out doors in an open space, where a distance of 500 feet should be measured off. At the spot from which the distance is measured the observer with the spy-glass should stand; at the spot 500 feet away the support should be placed.

By means of clamps fasten to the support a pendulum of such length that it will *beat* half seconds. (See your record of Exp. 70.) That the bob may be seen as it swings back and forth, pin a piece of white paper about

20^{cm} square on the upright behind the pendulum. The student beside the support starts the pendulum swinging, taking care to have it swing only a little way from side to side; if it swings off the paper, the observer looking through the spy-glass cannot see the pendulum. When the pendulum reaches one extremity of its swing, the student standing beside the support strikes the boards together. If the sound reaches the ear of the student looking through the spy-glass, at the instant in which the pendulum reaches the other extremity of its swing, then it has taken the sound just 0.5 second to travel 500 feet. In order that the student at the spy-glass may have a good many opportunities to observe the pendulum and listen to the sound, the student beside the support should strike the boards together every time the pendulum gets to the end of its swing next to him. By moving the spy-glass from or towards the pendulum, the distance that the sound will travel in 0.5 second can be found.

Interchange the spy-glass and the pendulum; then repeat the experiment, the student who before looked through the spy-glass now taking his position beside the support. For the velocity of sound in air, take the average of the two distances found.

From your observations on the velocity of sound per 0.5 second, what is the velocity of sound per second in feet?

QUESTION. If there are $15{,}240^{cm}$ in 500 feet, what is the velocity of sound per second in centimeters?

Experiment 73. *To find whether the air is necessary for the transmission of sound.*

Apparatus. A Kjeldahl flask; a one-hole rubber stopple to fit the flask; a piece of pressure tube about 30^{cm} long; a glass tube long enough to reach to the middle of the body of the flask; a little toy bell; an air-pump.

Directions. Push the glass tube through the stopple, and over the end that goes into the flask slip a bit of rubber tube about 0.5^{cm} long. With a bit of thread fasten the bell to the glass tube; the bit of rubber will keep the thread from slipping off. Over the other end of the glass tube slip the pressure tube. Put the stopple in place. Shake the flask, and listen to hear the bell ring. Connect the flask to the air-pump by means of the rubber tube. Pump out a little air, and shake the flask.

Is the sound of the bell fainter than before?

Pump out some more air.

On shaking the flask, can you hear the bell?

What inference can you draw from this experiment?

Point out in what way you have used the *method of differences* in this experiment.

Can sound pass through a vacuum?

Experiment 74. *To find whether a musical sound can be produced by a vibrating body beating the air at regular intervals.*

Apparatus. A straight piece of clock-spring about 50^{cm} long; a vise.

Directions. Fasten the clock-spring in the vise with about 45^{cm} projecting horizontally. Set the spring vibrating horizontally through a small arc, and record the number of vibrations made in 0.5 minute. Then set the spring swinging through a large arc, and record the number of vibrations made in 0.5 minute.

Does the time of vibration of the spring depend upon the amplitude of vibration?

Push the spring farther into the vise leaving about 20^{cm} projecting. If possible, make observations and record as before.

What is your inference?

Leave only about 8^{cm} projecting, and finally only about 4^{cm}.

When the spring now vibrates, does it give a musical sound?

Does the pitch become higher or lower as the number of vibrations increases?

From the results of this experiment, should you say that a musical sound has been produced by a vibrating body beating the air at regular intervals?

TUNING-FORK.

66. The Tuning-Fork. The tuning-fork is an acoustic instrument of great interest. The spring of the preceding experiment serves as a starting-point in the description of the tuning-fork, which may be looked upon as an elastic bar bent into a U-shape, free at both ends, and supported in the middle where the stem or handle is inserted. Tuning-forks are usually made of steel. The tuning-fork is excited (set in vibration) by striking the outside of one of the prongs against a board covered with leather or flannel, but never against a table or other hard object. Shortly after the fork has been excited, its tone becomes pure and simple.

NOTE. The tuning-fork was invented in 1711, by John Shore, a trumpeter in the service of George I. of England.

TRANSMISSION OF SOUND.

67. Mode of Transmission of Sound in Air. It was learned in Exp. 73 that the presence of air was necessary for the transmission of sound from a bell to the ear. But an inquiry of importance is, how the sound is transmitted by the air, by a *puff*, that is, by a small gust of air in which the air particles are sent from the sounding body to the ear, or by a *pulse*, that is, to-and-fro motion, of the air particles. The object of the next experiment is to satisfy this inquiry.

Experiment 75. *To find whether the movement of air, when sound passes through it, is of the nature of a puff or of a pulse.*

Apparatus. A long tin tube that tapers at one end; a candle; touch-paper.[1]

Directions. Place the tube in a horizontal position on the table. At the small end put the lighted candle (see Fig. 47), carefully shielded from air currents, with the flame opposite the opening and but 2^{cm} or 3^{cm} away. Get a student to strike together two blocks of wood at the large opening.

What happens to the flame?

Is the flame affected thus by a puff or by a pulse of air?

To answer this question, having filled the tube with smoke by burning touch-paper in it, strike the blocks together and watch the appearance at the small end of

[1] Touch-paper is made by soaking filter paper or blotting paper in a saturated solution of nitrate of potash, and then drying the paper.

the tube. The purpose of the smoke is to make visible to the eye the currents of air, if any, that are set in motion by clapping the blocks together. To see what

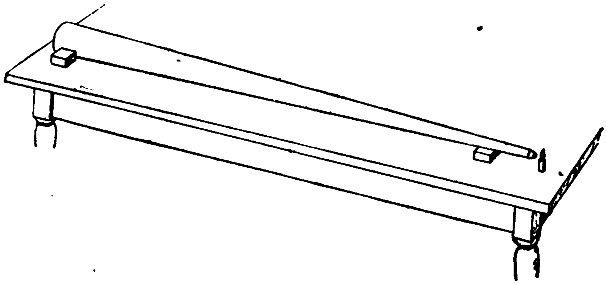


FIG. 47.

appearance a *puff* of air would produce, let the student at the larger end of the tube blow into the tube with a quick short *puff*.

From the behavior of the smoke when the blocks are struck together, should you say that the candle had been affected by a puff of air or by a pulse?

68. Is Sound a Wave-Motion of Air? From Exp. 73 you have already learned that the presence of air is necessary for the transmission of sound, and from Exp. 75 you have seen that when conveying sound, the particles of air have a back-and-forth motion. Can this motion be a wave-motion of air? To prepare yourself for answering this question, recall your study of water waves and the idea you gained of what is meant by interference (see pages 156 and 157). By the interference of two sets of water waves, there were produced waves of different sizes, from very small ones up to waves of greater magnitudes

than those of either of the original set of waves; calms were also produced. If sound is really due to a wave-motion of the air, phenomena of interference ought to be observed under suitable conditions. In the interference of sound waves, silence would correspond to a calm in the interference of water waves, and varying degrees of loudness of sound to water waves of different heights. The object of the next experiment is the study of sound under different phases, with a view to determining whether phenomena of interference can occur.

Experiment 76. *To find whether the phenomena of interference can be observed in sound.*

Apparatus. A tuning-fork; a 100^{cc} graduate; a cylinder of cardboard about as long as one of the prongs of the tuning-fork and about 2^{cm} in diameter; a tuning-fork, high C.

Directions. By striking one prong of the tuning-fork against a piece of leather laid upon the table, set the fork in vibration. Then hold the fork above the graduate in such a position that the ends of the prongs lie in the axis of the graduate. Pour water slowly into the graduate. As the water rises, the sound grows louder; and then as more water is added, the sound grows weaker. In this experiment adjust the level of the water till the sound of the fork is strongly reinforced. Turn the fork slowly round its axis.

For certain positions of the fork, do the sounds become nearly inaudible?

If any such positions are found, hold the fork steadily in one of them, and then carefully slide the cardboard cylinder over one of the prongs, as shown in Fig. 48,

without allowing the cylinder to touch either prong, as that would interrupt the vibrations.

Is the sound restored?

Could you account for what you observe by regarding each prong of the fork as sending out sound, but as one of the prongs is nearer the mouth of the graduate than the other, the sound reflected by the surface of the water in the graduate interferes with the sound from the more remote prong?

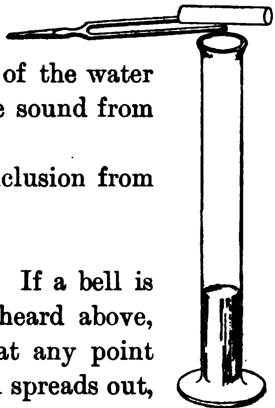


FIG. 48.

On the whole, what is your conclusion from this experiment?

69. Form of a Sound Wave. If a bell is sounded, the tone of the bell is heard above, below, at one side, and, in fact, at any point the ear may be placed. The sound spreads out, then, in spheres whose common center is the position of the bell. The sound wave is made up of a condensed portion (where the particles are crowded together), corresponding to the crest of a water wave, and of a rarefied portion (where the particles of air are farther apart than usual), corresponding to the trough of a water wave. Where a condensed portion of a sound wave coincides with the rarefied portion of another equal wave, silence results; where the condensed portion coincides with condensed portion, and rarefied portion coincides with rarefied portion, a sound louder than that due to either wave is produced.

Between what points do you measure the length of a sound wave?

The diagram (Fig. 49) represents the concentric waves of sound spreading from the point of disturbance at the center. Where the circles of dots are nearest together, there is the condensed portion of the wave; where the circles of dots are farthest apart, there is the rarefied portion of the wave.

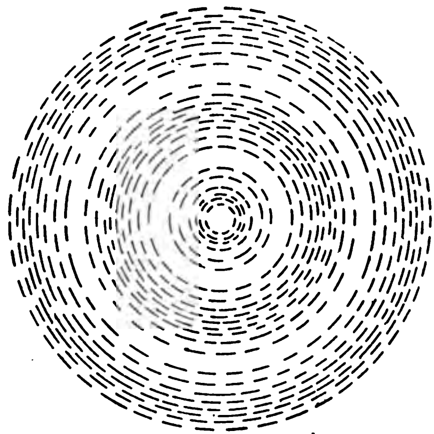


FIG. 49.

In the propagation of water waves the motion of the water particles is to and fro at right angles to the direction in which the wave moves.

In the case of sound waves in air, do the air particles move to and fro at right angles to the direction in which the sound travels?

RAPID VIBRATIONS.

70. Method of Counting Rapid Vibrations. In the experiment with the clock-spring clamped in the vise, you saw that by shortening the spring the musical sound emitted increased in sharpness, or pitch, and you also observed that the vibrations became so rapid that the eye could not follow them quickly enough to count them. In the next experiment we shall find how many vibrations per

second a certain tuning-fork makes. In this experiment a piece of smoked glass is drawn beneath a vibrating pendulum (Fig. 50) and a vibrating tuning-fork. A little

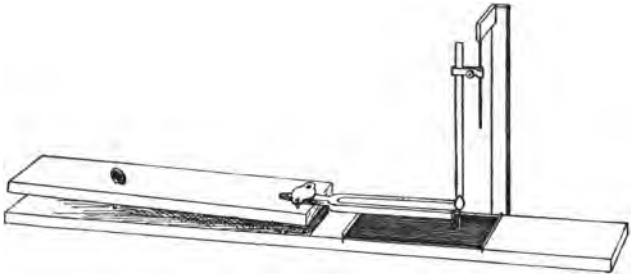


FIG. 50.

style, made of a bristle, is fastened to the pendulum and another to the fork. These styles trace out curves on the glass as shown in Fig. 51. The wavy line is traced by

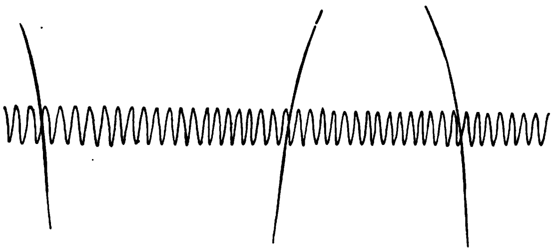


FIG. 51.

the style attached to the tuning-fork, while the other lines are traced by the style attached to the pendulum.

Experiment 77. *To find the number of vibrations made in a second by a tuning-fork.*

Apparatus. A tuning-fork, middle C; a piece of apparatus provided with a support for the tuning-fork and the pendulum; a watch; a bass-viol bow; rosin; a rectangular piece of glass.

Directions. Clean the glass, and then slightly smoke it by holding it in an ordinary flame. Lay the glass on the carrier beneath the pendulum. To the little rod projecting from the lower side of the pendulum bob fasten, by means of thread, a flexible bristle whose end shall bear lightly on the surface of the glass, so that, as the pendulum swings, a line shall be traced in the thin layer of soot which covers the glass. Near the end of one prong of the tuning-fork a somewhat stiffer bristle should be fastened by a bit of wax. This bristle should lightly touch the glass, to whose surface it should be somewhat inclined. Find how many vibrations the pendulum makes in a minute.

How long does it take the pendulum to make one vibration?

Now place the tuning-fork, mounted on its support, in a position to bring the end of the bristle attached to it as close as practicable beside that attached to the pendulum. When both pendulum and fork are vibrating, the bristles should move parallel to each other. When all the adjustments are made, set the pendulum swinging, excite the fork by drawing the bow, which has been well rosined, across it; then draw the carrier holding the glass plate along at right angles to the direction in which the pendulum and fork are vibrating.

Remove the glass, count carefully, using a magnifying glass if necessary, the number of vibrations recorded by the fork on the glass between the points corre-

sponding to one vibration of the pendulum. Record this number.

We already know the time the pendulum takes to make one vibration, and we have just found the number of vibrations the fork makes in an equal length of time.

How many vibrations does the middle C tuning-fork make in a second?

TUNING-FORK AND RESONATOR.

71. The Determination of the Velocity of Sound in Air by means of the Tuning-Fork and Resonator. If we know the number of vibrations which a tuning-fork makes in a second and the length of a column of air that will reinforce the tone of the fork as in Exp. 76, we can compute from these data the velocity of sound in air. The object of the next experiment is to obtain the data and make the computation.

Experiment 78. *To find, by means of a resonance tube, the velocity of sound in air.*

Apparatus. A glass jar 35^{cm} or more in depth, and from 3^{cm} to 10^{cm} in diameter; a tuning-fork, middle C; a thermometer.

Directions. Place the jar on the table, and over its mouth hold the vibrating tuning-fork with the ends of its prongs in line with the axis of the jar. Pour water gradually into the jar until the sound of the fork is strongly reinforced. By pouring in a little more water, or by pouring out a little, get the length of the air column from the mouth of the jar to the surface of the water such that the sound will be the very loudest. Then measure and

record in centimeters the distance from the mouth of the jar (Fig. 52) to the level of the water; also record the diameter of the jar in centimeters.

Instead of pouring water into the jar, the level of the water may be readily changed by means of a siphon, consisting of two glass tubes of small bore joined by a rubber tube. One of the glass tubes is bent so that it will reach nearly to the bottom of the jar when hung over its edge, the other glass tube is bent to hang over the edge of a jar containing a supply of water. The rubber tube is long enough to allow the jar containing the supply of water to be raised or lowered so that water will flow into the other jar or out of it, thus raising or lowering the level.

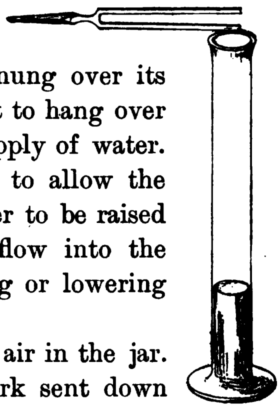


FIG. 52.

Record the temperature of the air in the jar.

The forward motion of the fork sent down the tube an impulse which, at the surface of the water, was reflected in time to reinforce the impulse given by the fork in its backward motion. Thus during the forward motion of the fork, that is, during half a vibration, the sound traveled to the water and back. When the sound passes through the mouth of the jar, a spreading of the sound occurs, so it is found necessary to add to the distance which the sound travels in one-fourth of a vibration (that is, to the distance from the mouth of the jar to the level of the water) one-fourth of the diameter of the jar.

After making this correction, what distance do you find that sound travels during one vibration of the fork?

From the experiment you have already performed, how many vibrations does the tuning-fork make in a second?

From your data, what is the velocity in centimeters per second of sound in air?

72. Sympathetic Vibrations. In the last experiment the fork was able by its vibrations to set into vibration a column of air of definite length, and thereby the loudness of its note was greatly increased. The air column, when of proper length, *vibrated* in *sympathy* with the fork; hence such vibrations are called *sympathetic vibrations.*

In most sonorous bodies (for example, a tuning-fork) mechanical movement (the motion of the prongs of a tuning-fork, for instance) is transformed into sound. It would be interesting to inquire whether this process has ever been reversed; whether, in other words, sound vibrations can generate mechanical motion. There is a little instrument, known as a sound radiometer, or an acoustic reaction wheel, which can be kept in motion by sound. It is made of four small tubes open at one end. These tubes, made of aluminum on account of its lightness, are accurately tuned to the same note. Two light rods or wires are fastened together at right angles, making four arms of equal length. A tube, or resonator, is fastened at the middle of its length to the extremity of each arm in such a way that its axis lies in a plane below but parallel to that of the crossed wires. The whole is delicately poised on a pivot in a horizontal position. When this instrument is put near a fork giving the same note as that to which the resonators are tuned, the little wheel begins to rotate,

and continues in rotation as long as the tuning-fork continues in vibration.

73. Beats. If two tuning-forks, one of which makes 254 vibrations while the other makes 255, are set in motion, a peculiar palpitating effect results, produced by bursts of sound, separated from one another by intervals of comparative silence. These bursts of sound are called *beats*. By the principle of interference (see Exp. 76) the production of beats can be fully explained. Suppose the forks to be in vibration; if we start from the time when the condensed portion of the waves from each fork reaches the ear at the same instant, that is, when the sound is loudest, just one second will elapse before the sound is loudest again. The fork making 255 vibrations per second gains a vibration in one second over the other fork, that is, in one second the note from this fork gains one wave-length; but since at the middle of the second it has gained only half a wave-length, the rarefied portion of the sound wave from this fork combines with the condensed portion of the sound wave from the other fork, the two portions neutralize each other, and silence results. During every second, then, that passes while the forks are vibrating together, there will be one beat and one period of silence. If one fork had made 254 vibrations per second and the other 256, two beats and two periods of silence would have occurred during the second.

If, when two tuning-forks are sounding together, there are beats, the number of beats per second tells the difference between the number of vibrations per second of the two forks; further, if the number of vibrations per second

of one of the forks is known and it is also known which of the forks gives the higher note, it is possible to find the number of vibrations per second made by the fork whose number of vibrations is unknown. For example, two forks *A* and *B* are sounded together, and four beats per second are counted. If it is known that *A* makes 256 vibrations per second, and that its note is lower than that of *B*, it follows that *B* makes in one second 260 vibrations.

74. Octave; Concord; Discord. Two notes are an octave apart when one is produced by twice as many vibrations as the other. Thus a tuning-fork that makes 508 vibrations per second emits a note which is the octave of the note given by a fork vibrating 254 times per second. When two notes an octave apart are sounded together, the result is pleasing to the ear, and there is said to be *concord*. Besides notes an octave apart, there are others that produce concord when sounded together; but there are many notes which, on being sounded at the same time, produce a disagreeable impression on the ear; such notes are said to produce *discord*. The unpleasant, jarring effect of discord is due to the production of beats.

VIBRATING STRINGS.

75. Pitch of Vibrating Strings. It will be the purpose of the three following experiments to find what relations exist between the length, the tension, the thickness of a stretched wire, and the number of vibrations per second.

Experiment 79. *To find the relation between the length of a stretched wire and the number of vibrations per second.*

Apparatus. A spring balance of 30 pounds' capacity; a piece of spring brass wire, No. 22 B. & S., 1.5^m long; two of the triangular pieces of wood used in Exp. 62; a meter stick; a middle C fork, and another an octave higher.

Directions. Fasten one end of the wire to a screw in the table-top, and lay the wire straight on the table; fasten the other end to the balance. Hook the ring of the balance to the screw for applying tension, as shown in Fig. 53. (For Exps. 79 and 80 consider only the bal-

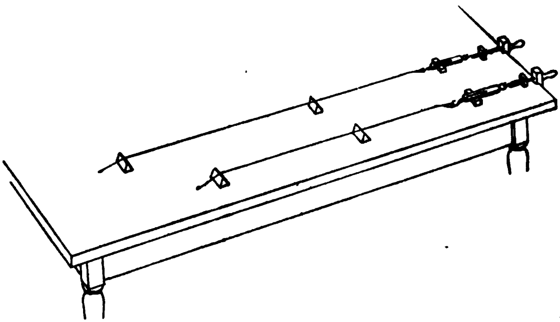


FIG. 53.

ance and wire which, in the figure, is next to the edge of the table.) Under the wire near the screw fasten by a brad one of the prisms with its edge at right angles to the wire. Under the wire lay the other prism parallel to the first. If the balance, as it should, holds the wire about as high as the top of the movable prism, this prism can be moved along under the wire without changing the tension to any extent.

Allowing for the zero error of the balance, when used in the horizontal position, stretch the wire with a force of 20 pounds, and, keeping this tension constant, find the

lengths that will give notes corresponding to the two forks respectively. Set the wire into vibration by plucking it in the middle, and, with the ear close to the wire, listen for the *fundamental* note, which may for an instant be obscured by harsh or grating overtones.

A student who has no keen perception of musical pitch can secure nearly perfect unison by sounding the wire and the fork at the same time; the beats, which become very apparent when the sounds are near unison, will guide him in his judgment. It will be well to press the wire with the finger lightly against the movable prism so as to limit the vibrations to that part under consideration.

One fork is an octave higher than the other.

Divide the greater number of vibrations by the smaller; also the greater length of wire between the prisms by the smaller.

Are the quotients equal?

The inference you can draw we shall call Law 1.

State Law 1.

Questions. If a string 100^{cm} long gives a certain note when plucked, what must be its length to give the note an octave higher? What must be its length to give the note an octave lower?

Experiment 80. *To find the relation between the tension of a wire and the number of vibrations per second.*

Apparatus. The same as in Exp. 79, without the higher fork.

Directions. Correcting for the zero error of the balance, when in the horizontal position, make the tension 5 pounds. Find what length of wire will give a note whose pitch corresponds to that of the middle C fork.

Turn back to your record of Exp. 79; you ought to find there a length recorded equal or nearly equal to the one just found.

To which fork does the note of the recorded length of wire correspond?

Divide the number of vibrations of this fork by the number of vibrations of the fork used in the present experiment; also divide the tension used in the last experiment by the tension used in this experiment.

If the two quotients are not equal, try to make the second equal to the first by squaring or by taking the square root.

The relation thus found between the number of vibrations per second and the tension is called Law 2.

State Law 2.

QUESTIONS. If a wire under a tension of 7 pounds gives a certain note, how much higher would the note become on increasing the tension to 28 pounds? How much lower would the note become if the tension were reduced to 1.75 pounds?

Experiment 81. *To find the relation between the thickness of a wire and the number of vibrations per second.*

Apparatus. A piece of spring brass wire, No. 28 B. & S., stretched as shown in Fig. 53; the fork of higher pitch.

Directions. Correcting as before for the zero error of the balance, when used in the horizontal position, apply to the wire a tension of 5 pounds. Find what length of wire will give a note whose pitch corresponds to that of the fork of higher pitch.

Turn back to Exp. 80; you ought to find a length recorded equal or nearly equal to that obtained in this experiment.

Divide the number of vibrations of the wire of this experiment by the number of vibrations of the wire of Exp. 80. The ratio of the thicknesses of the two wires is as 2 to 1.

If the ratio of the thicknesses is not equal to the ratio of the number of vibrations, try to make it equal by squaring or taking the square root, inverting if necessary.

The relation thus obtained between the thickness of the wire and the pitch is called Law 3.

State Law 3.

QUESTIONS. If a wire 1^m long gives a note of a certain pitch, how much higher will be the pitch of a note given by a wire, of the same material as the first, of equal length and stretched by the same tension, but of only one-half the thickness? How much lower will the note be if the wire is twice as thick?

NOTE. The strings or wires which we have considered are supposed to be of the same material. Whenever the material of two strings, alike in all other respects, differs in density, there is a fourth law, "The number of vibrations is inversely proportional to the square roots of the densities."

76. Loudness; Pitch; Quality. The *loudness* of a note depends upon the amplitude of vibration. When a tuning-fork is set in vibration, the note, loud at first, gradually dies away, becoming fainter and fainter as the amplitude of the fork's vibrations decreases. The *pitch* of a note depends not upon the amplitude of vibration, but upon the number of vibrations made in a given time: the greater the number of vibrations, the higher the pitch; the smaller the number, the lower the pitch. The *quality* of a note depends neither upon the amplitude of vibration nor upon the frequency of the vibrations, but upon the peculiar tones which accompany the production of the fundamental note. On hearing C sounded on a piano

and then on a violin, the ear perceives that the pitch is the same, yet it distinguishes between the note emitted by the piano and that given by the violin; a note sounded on a piano has a different quality from that of the same note sounded on a violin. To illustrate the modification the fundamental note undergoes by the peculiar tones, depending upon the kind of musical instrument used, let the student imagine, what is so often seen at the seashore, long swelling waves coming from the sea, whose surface except for these waves is unbroken. These waves may be taken to represent the fundamental note. Now suppose a light breath of air disturbs slightly the surface of these waves, which become dimpled. The wavelets thus produced modify to a very limited extent the character of the original waves. So with musical instruments, each instrument gives out the fundamental note, C for example, which is the same for them all; but each instrument by reason of peculiarities of its construction gives out little notes, faint to be sure, but sufficient, nevertheless, to modify the fundamental note and give to it an appearance or quality different from the corresponding note of some other instrument.

THEORY OF SOUND.

77. Sketch of the Development of the Theory of Sound. More than two thousand years ago Pythagoras[1]

[1] Pythagoras (pronounced *py-thag'o-ras*) (about 569–500 B.C.) founded a school of philosophy whose members in honor of their teacher were called Pythagoreans (pronounced *pў-thăg-o-rē'ans*). This school busied itself with many fantastic mathematical and philosophical speculations,

invented the monochord, an instrument similar to that used in the last three experiments. With this instrument Pythagoras made several discoveries about the sounds produced by a stretched string when vibrating. One of his first discoveries was that a string which gives a certain note will give a note an octave higher, if the string is made one-half as long.

Very little advance was made from the time of Pythagoras till that of Mersenne,[1] who proved experimentally that the number of vibrations is inversely proportional to the length of the string; that the number of vibrations of a string is proportional to the square root of its tension; and that the number of vibrations is inversely proportional to the thickness of the string.

The laws of vibrating strings have been determined mathematically as well as experimentally. It was Lagrange[2] who completed the work from the mathematical point of view at which his predecessors had labored so industriously.

It was reserved, however, for Helmholtz, a very eminent physiologist, physicist, and mathematician, to lay the foundation of musical science, which he accomplished about the middle of the nineteenth century.

the most famous of which was the doctrine of "the harmony of the spheres." According to this doctrine, the heavenly bodies in their motion through the sky give out grand and wonderful music, but so fine and delicate that our ears, accustomed to the gross sounds immediately around us, are deaf to this "music of the spheres."

[1] Mersenne (pronounced *mĕr-sen′*) (1588–1648) was a Franciscan friar. He has been called the "Father of Acoustics."

[2] Lagrange (pronounced *lä-grŏnzh*) (1738–1815) was a celebrated French mathematician.

LAWS.

78. Laws of Nature; Theory. The *laws of nature* are general truths which have been found by diligent search among the facts obtained by observation and experiment. For example, that "the deflection of a rod is proportional to the load" is a law, or general truth, which all rods, provided the load is not too great, obey. A law of nature, it must be remembered, differs from a law for the government of society; the former is fixed and changeless, while the latter lasts only till men see fit to repeal or amend it.

From the results of your work in the laboratory, name fifteen laws which you have inferred. If now you go through the process of reasoning by which you arrived at these laws, is there any instance in which you did not infer a law from particular cases? For the sake of illustration take the law just quoted; by reference to your record you will see that there were from twelve to sixteen particular cases among which there seems to be this bond, or uniform relation, "the deflection of a rod, in each case, is proportional to the load." Life would be far too short to test every rod to see if this relation holds, so from his experience in a few particular cases the student infers it to be true in all. The belief in the truth of his inference is strengthened by the answers given by nature to other inquiring students. The student must not suppose that the confidence of men of science in the true statement of the laws of nature rests on inferences from data so imperfect as those obtained in our own laboratory work. Our data may *suggest* the possibility

of the law, but the physicist is not satisfied with this; he makes accurate measurements, he varies the different cases, he performs different experiments, till he accumulates a large amount of evidence from which to draw his inferences.

When we speak of the *theory of sound*, we mean the general and accurate knowledge of the laws of sound, just as when we speak of the theory of quadratic equations we mean the general and accurate knowledge of the laws which connect the coefficients and the constant term with the roots.

The term *theory*, however, is ambiguous; sometimes it has the meaning just given, at others it is synonymous with the term *hypothesis*.

FALLACIES.

79. Fallacies of Observation. In Art. 29 the attention of the student was called to the importance of distinguishing between facts and inferences. In this matter too much attention and care cannot be given to training the mind to careful habits of discrimination. With the best of intentions of telling the truth in a court of law, a witness with little knowledge and little mental cultivation, when undertaking to give an account of simple occurrences that he has seen, often mingles facts and conjectures in such confusion, that the lawyer only by skillful cross-examination and a careful sifting of the evidence can make the witness separate the facts and the inferences (false or true) which he has drawn from these facts.

Even an acute and well-trained mind is not always free from mistaking an inference for a direct perception. An amusing instance of this is related of Dr. Wollaston, a celebrated English chemist. When Sir Humphry Davy placed in his hand for inspection the scientific curiosity of the day, a bit of potassium, a substance so light that it will float on water, Dr. Wollaston carefully examined the potassium, noted its metallic lustre, and did not hesitate to declare it a metal. In this philosopher's mind intimately associated with the notion of metal was also the notion of weight, and the evidence of his sense of touch was insufficient to separate the two ideas; so, balancing the specimen on the tips of his fingers, he exclaimed, "How heavy it is!" He mistook his judgment of the weight of the substance for the sensation itself.

EXAMPLES.

1. The length of the seconds pendulum at Greenwich is 99.413^{cm}; find the length of a pendulum which makes a single oscillation in 1.5 seconds.

2. A tuning-fork makes 256 vibrations per second, and the velocity of sound in air is 340^{m} per second; what is the wave-length of the note produced?

Solution. If an observer be at a distance of 340^{m} from the fork, there will be, between the fork and the ear, 256 condensations and 256 rarefactions; but a condensation and a rarefaction make up a wave; so there will be 256 waves occupying a distance of 340^{m}, hence, if in 340^{m} there are 256 waves, the length of one wave will be $\frac{340}{256} = 1.33^{m}$.

3. Find the wave-length of a note making 1000 vibrations per second, both in air and in water; the velocity of sound in air being 1100 ft. per second, and in water 4900 ft. per second.

4. Taking 1120 ft. per second as the velocity of sound in air, find the number of vibrations which a tuning-fork, vibrating 254 times in a second, must make before its sound is audible at a distance of 144 ft.

5. A stretched string 10 ft. long is in unison with a tuning-fork making 256 vibrations per second; the string is shortened 4 ft.; how often will it now vibrate in a second?

6. A string is fastened at one end to a peg in a horizontal board, and the other end passes over a pulley and carries 16 pounds. The string thus stretched gives the note C. What weight must be put in place of the 16 pounds, so that the string shall give the next lower octave?

7. Find the distance of an obstacle which sends back the echo of a sound to the source in 1.5 seconds, when the velocity of sound is 1100 ft. per second.

Solution. In 1.5 seconds, sound travels $1100 \times \frac{3}{2}$, or 1650 ft.; this distance the sound travels in going to the obstacle and in returning, hence the distance of the obstacle is 825 ft.

8. The distance from the top of a well to the surface of the water is 210 ft. What time will elapse between producing a sound at its mouth and hearing the echo? (Velocity of sound = 1100 ft. per second.)

CHAPTER V.

LIGHT.

80. Self-Luminous Bodies; Non-Luminous Bodies. When we are in a place exposed to either the sun or any other glowing body, we become aware of the existence of objects around us. If the place be securely shielded from the sun, or if the glowing substance be quenched, then by the eye we perceive nothing, all is blank; but when the substance is kindled again, or the sun shines in once more, then the sight again perceives the objects which were but a moment before invisible. This mysterious something that is necessary to render objects visible is called light. Bodies like the sun or a lighted lamp, therefore, have the property of rendering visible not only themselves, but also the objects on which their light shines. Bodies which have this power are called *self-luminous bodies*. On the other hand, bodies which require the presence of a self-luminous body to enable us to see them are called *non-luminous bodies*.

81. Transparent, Translucent, and Opaque Substances. If, in a lighted room, we hold a piece of window-glass before the eyes, we readily see the objects in the room through the glass. Substances, like glass, through which objects can be distinctly seen are called *transparent*. Substances, like oiled paper, through which, though light passes, objects cannot be distinctly seen, are called *translucent*.

Substances, like iron, through which no light passes are called *opaque*.

Make a list of four transparent substances, another of four translucent substances, and a third of four opaque substances.

PHOTOMETRY.

82. Intensity of Illumination. When reading a book in the evening, you will find it more and more difficult to see the letters the farther the book is carried from the lamp. In other words, on carrying a book from the lamp, the degree to which the lamplight illuminates the page is diminished. It will be interesting, as an introductory experiment in light, to find the relation between the degree to which a lamp or a candle illuminates a given object, as a screen, and the distance of the object from the lamp. In the course of the experiment we shall have occasion to use the term *intensity of illumination*, and it is important to understand clearly its meaning, so we give the following definition:

Definition. *By the intensity of illumination is meant the degree to which a source of light supplies a given body with light.*

Experiment 82. *To find the relation between the intensity of illumination and the distance.*

Apparatus. A Letheby's photometer; five candles. Letheby's photometer (Fig. 54), an instrument for measuring the intensity of illumination, is constructed as follows: there is a long bar or rod on which a screen of paper, stretched on a frame, is placed in a vertical position. This screen has a translucent spot of paraffine on

its center. Two mirrors, one for each eye, are so placed that an observer may see both sides of the screen at the same time. One of the lights is placed on one side of the screen; the other, on the other

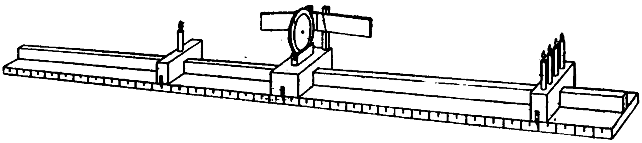


FIG. 54.

side. The mode of action of the instrument depends on the fact that when a piece of paper having a paraffine or other grease spot on it, is equally illuminated on both sides, the spot becomes nearly, if not quite, invisible.

Directions. This experiment is to be performed in a darkened room. Place in line four candles, all of the same height, on one of the sliding blocks, in such a way that *the line of candles is at right angles to the bar.* On the other side of the screen place on a sliding block the other candle. Light the candles and trim the wicks so that the flames shall be of equal size. In order to let the candles get well burning, it is best to wait a few minutes before making any measurements. When trimming the candles, look at them not with the naked eye, but through colored glasses; otherwise the eye will not be as sensitive to the difference of light and shade on the screen. Slide the block carrying the four candles along the bar till the middle of the line of flames is at a distance of 80^{cm} from the screen. By sliding the single candle back and forth along the bar, find a position for it such that the spot shall disappear as you look into the mirrors (or, if the spot cannot be made to vanish entirely, get the two images of

the spot, as seen in the mirrors, of the same shade). Record the distance of the four candles from the screen; also the distance from the screen of the single candle when placed in the position you have been directed to find.

Next put the four candles at a distance of 160^{cm} from the screen, and find the corresponding position of the single candle. As before, record the distances.

Assuming that each of the five candles gives out the same amount of light, how does the amount of light given out by the four candles compare with that given by the single candle?

In each case, how does the distance of the four candles compare with that of the single one?

In this experiment the intensity of illumination on the screen due to the single candle was equal to the intensity of illumination due to the group of four candles placed at a greater distance from the screen on the other side. Now consider that from the group of four candles, when the distance has been properly adjusted, three are removed, so as to leave only one candle; how much light falls upon the screen from this single candle as compared with the amount of light from the group of four?

Divide the greater distance (the distance of the four candles from the screen) by the lesser distance (the distance of the single candle on the other side of the screen); also divide the intensity of illumination of the single candle by the intensity of illumination of a single candle supposed to be placed at a distance from the screen equal to that of the group of four candles.

If the two quotients are not equal, try to make them by squaring or cubing one of them.

What relation should you infer holds between the distance of a light from a screen and the intensity of its illumination?

QUESTIONS. How would the distances compare if 9 candles were used in place of the 4? If 16 candles were used? If 25 candles were used?

Experiment 83. *To find how many candles will give the same intensity of illumination as a kerosene lamp.*

Apparatus. A Letheby's photometer; a kerosene lamp with a chimney; a candle.

Directions. At a distance of 50^{cm} on one side of the screen place the candle. On the other side of the screen place the well-trimmed lamp with its flame turned flatwise towards the screen. Light the candle and wait till it is burning well. See that the lamp-flame and the candle-flame are at equal distances above the table. Move the lamp till the spot disappears or till both images are of the same shade. Measure and record the distances.

Denote the power of the candle by 1, and that of the lamp by x, then by the law brought out in the preceding experiment:

$$\frac{1}{(\text{distance of the candle})^2} = \text{intensity of illumination of}$$
the candle.

$$\frac{x}{(\text{distance of the lamp})^2} = \text{intensity of illumination of}$$
the lamp.

But in the present experiment you have adjusted the distances in such a way as to make the intensity of illumination of the candle on the screen equal to that of the lamp, hence,

$$\frac{x}{(\text{distance of the lamp})^2} = \frac{1}{(\text{distance of the candle})^2}$$

$$\therefore x = \left(\frac{\text{distance of the lamp}}{\text{distance of the candle}}\right)^2.$$

Making use of this relation, compute the number of candles to which the lamp is equivalent.

Definition. *Photometry is the art of measuring the intensity of light.*

RAYS.

Experiment 84. *To find whether light passes through the air in straight lines.*

Apparatus. A Bunsen burner arranged to give the luminous flame; three pieces of cardboard.

Directions. Set up the three cards, through each of which a pin-hole has been made, in such a way that the flame or a portion of it can be seen through the holes.

When the flame can be seen, are the three holes in the same straight line?

Does, then, light pass through the air in straight lines?

83. Ray; Beam; Pencil. A single *ray*, or line, of light (Fig. 55, 1) is represented by a straight line. A *beam* of light, that is, a bundle of parallel rays (Fig. 55, 2), is represented by several parallel straight lines. A *pencil* of light, that is, a group of rays converging to or diverging from a point, is represented by a group of converging or by a group of diverging lines. Fig. 55, 3, represents a *converging pencil* in which the rays proceeding from some

source of light on the left draw nearer together, so as to cross each other at the point *O*. Fig. 55, 4, represents a

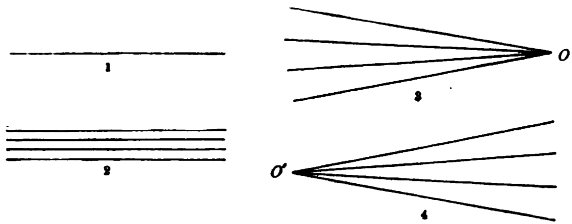


FIG. 55.

diverging pencil in which the rays proceeding from the source of light *O'* spread away from each other as they proceed to the right.

SHADOWS.

Experiment 85. *To find the cause of a shadow.*

Apparatus. An opaque cylinder 8^{cm} or 10^{cm} in diameter; two candles; a piece of cardboard about 20^{cm} square.

Directions. This experiment is to be performed in a darkened room. Place the cylinder on a table. Put the two lighted candles on the table at a distance of 10^{cm} or 12^{cm} from the cylinder, making the distance between the candles equal to the diameter of the cylinder. On the other side of the cylinder and about 10^{cm} from it support the piece of cardboard in a vertical position. Notice a black band bordered by two lighter ones on the cardboard. With a pin pierce a hole through the screen where the black band crosses. Look through this hole towards the lights.

Do you see either or both of the candles?

Pierce a hole through the screen where one of the lighter bands crosses. Through the hole thus made look towards the candles.

Can you see either candle?

Pierce a hole through the other lighter band.

Looking through this hole, which candle can you see?

Move the screen back and forth to see how the widths of the bands change.

As you move the screen away, what change is there in the width of the black band?

Pierce a hole through a brightly lighted part of the screen.

Looking through this hole, can you see more than one candle?

The black part of the shadow is called the *umbra;* the border is called the *penumbra.*

From your observations explain the formation of a shadow, accounting for the umbra and the penumbra.

Does the shadow extend from the cylinder to the screen?

IMAGES.

84. Formation of Images by Means of Small Apertures. If a hole is made in the shutter of a dark room, an inverted picture of the scene outside in front of the window appears on the wall of the room opposite the shutter. When sunlight passes through the spaces between the leaves of trees, *circular* patches of light are seen on the ground; if, however, the sun should be partly eclipsed, the patches of light would be crescent-shaped. The object

of the following experiment will be to study the formation of images obtained when light passes through small apertures, like the hole in the shutter or the small spaces between the leaves of trees.

Experiment 86. *To find an explanation of the formation of an image* (picture) *on the screen of a pin-hole camera.*

Apparatus. A pin-hole camera, which consists of a box about 10^{cm} in width and depth, and 30^{cm} long, with one end closed and the other open. In the open end a frame, carrying a translucent screen, slides. In the center of the closed end is cut a rather large hole, which is covered by a piece of thin sheet brass, through which a small hole is pierced.

Directions. Slide the screen into the box, hold the brass-covered end towards a bright gas-flame in a darkened room, and look into the open end of the box towards the flame. At first have the box near the flame, then gradually take it farther away, always looking in, through the open end, at the picture on the screen. Also slide the screen back and forth.

Can you get a position of the screen such that the image of the flame is very distinct?

Explain, by a drawing, why the image (picture) is inverted. (See your inference from Exp. 84.)

QUESTIONS. Why are circular patches of light seen on the ground beneath the trees in summer when the sun is shining? When the sun is partly eclipsed, why are these patches crescent-shaped?

REFLECTION.

85. Reflection of Light; Angle of Incidence; Angle of Reflection. If the eye is in a proper position when sunlight falls upon a suitable surface, the calm surface of

a lake for example, the image of the sun can be seen in the lake. The rays of light from the sun strike the surface of the water, and some of them are bent back from the surface and so reach the eye. This bending back of the rays is called *reflection.* The rays that strike the surface are called *incident* rays (Latin *incidere*, to fall upon). The rays that are bent back from the surface are called *reflected* rays (Latin *reflectere*, to bend back).

If a perpendicular be erected to the reflecting surface, the surface of the lake in this case at a point where an incident ray falls upon it, the angle between the perpendicular and the incident ray is called the *angle of incidence;* the angle between the perpendicular and the reflected ray is called the *angle of reflection* (Fig. 56).

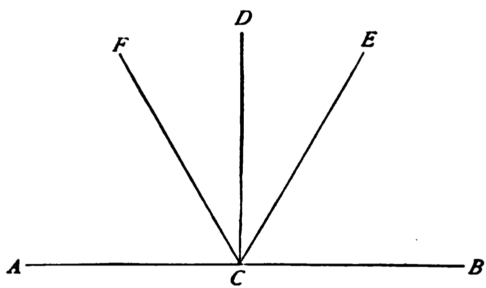


FIG. 56.

AB, reflecting surface; *CD*, perpendicular (in optics, often called a normal); *EC*, incident ray; *CF*, reflected ray; *ECD*, angle of incidence; *DCF*, angle of reflection.

Experiment 87. *To find whether there is any simple relation between the angle of incidence and the angle of reflection.*

Apparatus. A small plane mirror; a sheet of paper 50^{cm} square; a meter stick; a pin; a protractor; two rubber bands; a block.

Directions. By means of tacks at the corners, fasten the sheet of paper to the table. From the middle point of one side of the paper draw a straight line to the middle point of the opposite side. Fasten the mirror to a rectangular block, by means of two small rubber bands, so that the back of the mirror rests against one of the narrow sides of the block. Place the mirror thus arranged with its edge along the middle portion of this straight line. Have the silvered part of the mirror over the line. The reflection takes place at the silvered surface. At some distance in front of the mirror, but to one side of its center, stick a pin upright in the paper. On the paper lay the meter stick (Fig. 57) in such a position that its direction makes an acute angle with the face of the mirror. By sighting along the edge of the meter stick, point it towards the image of the pin formed in the mirror. When the meter stick has been carefully adjusted so as to point accurately towards the image, and at the same time to make an acute angle with the face of the mirror, draw, with a *sharp pencil*, a line on the paper along the edge of the meter stick to meet the mirror. Remove the mirror, and produce this line until it meets the line drawn across the paper. Draw a straight line connecting the point of meeting of the two lines with

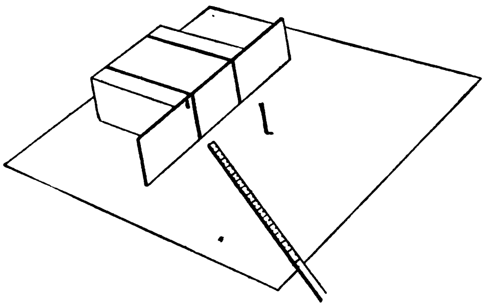


FIG. 57.

the pin. At the point of intersection of the three lines erect a perpendicular to the line drawn across the paper.

If the mirror were still in position, would the line last drawn be perpendicular to the surface of the mirror?

With the protractor, measure the angles formed by the oblique lines with the perpendicular. The angle formed with the perpendicular by the line from the pin is the *angle of incidence*, and the angle formed with the perpendicular by the line from the mirror to the eye (found by sighting along the meter stick) is the *angle of reflection.*

After measuring these angles with the protractor, can you infer a relation between the two angles?

If you have performed the experiment accurately, your answer to the question is the statement of the chief law of the reflection of light.

Make as brief a statement of the law as possible.

Preserve the paper and paste it into your note-book.

PLANE MIRRORS.

86. Images in a Plane Mirror. When you see your image, or reflection, as it is sometimes called, in a mirror, doubtless you have noticed that it appears behind the mirror; that when you move, the image moves. It will be the object of the two following experiments to find a relation between the distance of the object from the mirror in front and the apparent distance of the image behind the mirror; also something about the relative size and shape of image and object.

Experiment 88. *To find what relation holds between the position of the image of a point and the position of the point.*

Apparatus. The same as in the last experiment, with a fresh sheet of paper.

Directions. As in the preceding experiment, fasten the sheet of paper to the table, draw a straight line across the sheet, and place the mirror and the pin in position. Then lay the meter stick on the paper, and by sighting along its edge, which should be inclined at an acute angle to the face of the mirror, point it directly towards the image of the pin (the pin is so small that we shall consider its position as a mere point). With a sharp lead pencil, guided by the edge of the meter stick, draw a line on the surface of the paper directly towards the image. Without disturbing either the pin or the mirror, sight at the image of the pin from an entirely different direction, having this new direction make as large an angle as practicable with the former line along which you sighted, and as before draw a line towards the image. Now remove the mirror, and carefully produce the two lines you have drawn towards the mirror until they cross each other. Also from the pin draw a line at right angles to the line that the mirror rested on, and continue it till it crosses the other two lines.

Now replace the mirror in its old position, change the position of the pin, and make a new set of observations. Finally, put the pin in another position and make another set of observations.

Remembering that the *pin* is the *object*, and that its *reflection* in the mirror is the *image*, study the results of your experiment with a view to answering the following questions:

How far *behind* the mirror does the *image* appear to be

as compared with the distance of the *object in front* of the mirror?

For a given position of the object, does the image always appear at the same place, no matter from what direction you look into the mirror?

If a line is drawn from the object to the image, is this line perpendicular to the mirror?

Paste the paper into your note-book.

Experiment 89. *To find the relation between the size and position of an object and its image.*

Apparatus. The same as that of the preceding experiment, together with two pins and a fresh sheet of paper.

Directions. Fasten the paper to the table, draw a line across it, and on this line stand the mirror. At a distance of a few centimeters in front of the mirror draw on the paper a triangle (Fig. 58) whose sides are respectively 6^{cm}, 8^{cm}, and 10^{cm} long. By sticking three pins upright in the paper, mark the angles of the triangle. By the method of sighting used in the preceding experiment, find the position of the image of each of these pins; that is, find the positions of the angles in the image of the triangle. Connect by straight lines the points thus found.

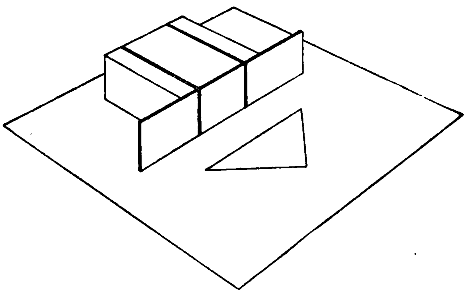


FIG. 58.

How does the size of the image compare with that of the object?

How does the distance from the mirror of any point in the image compare with the distance from the mirror of the corresponding point of the object?

As compared with the object, is the image inverted, that is, turned upside down?

SUGGESTION. Call to mind the images of the pins, whether they were inverted or upright.

Is the image laterally inverted, that is, is the right-hand side of the object opposite its own reflection?

Paste into your note-book the sheet on which the triangle is drawn.

Knowing that a *real* image can be caught on a screen, and that a *virtual* image cannot be caught on a screen, answer the following questions:

Is the image seen when looking into an ordinary looking-glass real or virtual?

What kind of image was formed in Exp. 86?

87. Multiple Reflections. When, after reflection at the surface of a plane mirror, a ray of light falls upon a second plane mirror, the ray is reflected from this second mirror in such a way as to make the angle of incidence equal to the angle of reflection. The result of the reflections of light from one plane mirror to another is a number of images, the number depending upon the angle which the two mirrors make with each other. The object of the next experiment will be to find the number of images formed when two mirrors are placed at different angles with each other.

Experiment 90. *To find the number of images formed by two plane mirrors, when making with each other* (a) *an angle of 90°*, (b) *an angle of 60°*, (c) *an angle of 40°.*

Apparatus. Two plane mirrors; two rectangular blocks of wood; four rubber bands; a sheet of paper 50^{cm} square; a protractor; a pin.

Directions. Fasten the paper to the table. By means of the rubber bands fasten the mirrors to the blocks. With a sharp lead-pencil draw on the paper near the center two straight lines crossing each other at right angles, that is, 90°.

Place the two mirrors (Fig. 59) in such a position that a long edge of one shall lie along one of these lines, a long

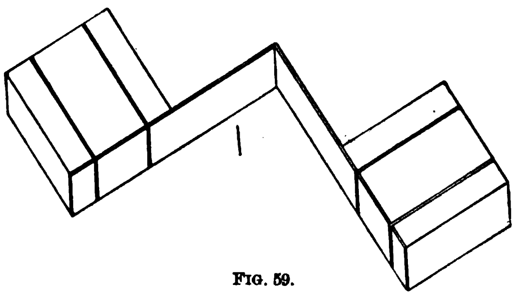


Fig. 59.

edge of the other mirror along the other straight line, thus making the angle formed by the planes of the mirrors a right angle. At a distance of about 4^{cm} from the place where the mirrors meet, stick a pin upright in the paper lying between the mirrors. Record the number of images of the pin.

Now by the aid of the protractor make the angle between the two mirrors 60°. To do this, draw a line on the paper, and then measure off with the protractor an angle of 60° from this line, and then draw a second line,

making an angle of 60° with the first; the mirrors can now be placed on these lines so as to make an angle of 60° with each other. Stick the pin into the paper as before, and count the number of its images. Record the number of images.

Finally, place the mirrors at an angle of 40° to each other. Record the number of images.

Experiment 91. *To find whether the multiple images formed by two plane mirrors lie on a circumference of which the intersection of the lines on which the mirrors stand is the center.*

Apparatus. The same as in the last experiment, together with a long pin.

Directions. Fasten the paper to the table; draw on it two lines at 60° to each other. Stick the little pin into the portion of the paper lying between the two mirrors, which should have been placed in position on the two lines. With the long pin locate the position of each of the images. These images appear to occupy positions behind the mirrors; by moving the long pin behind the mirrors this pin can be made to coincide with the position of each of the images. This position is secured when on moving the eye from side to side in front of the mirror the image and the long pin always occupy the same position. Mark each of these positions. Mark also the position of the object, that is, the pin stuck into the paper between the mirrors. Now clear the paper, and with the intersection of the two lines, the point over which the narrow edges of the mirrors came together, as a center, and a

radius equal to the distance of the object from the intersection of the two lines, describe a circumference.

Through what points, previously marked on the paper, does the circumference pass?

What inference can you draw from this experiment?

88. Dependence of the Number of Images on the Angle between Two Plane Mirrors. The relation that exists between the angle formed by two plane mirrors and the number of images of an object formed by the mirrors is as follows:

Provided the number of degrees contained in the angle between the two mirrors will divide without a remainder 360, the number of degrees in a circumference, the number of images formed will always be one less than the value of the quotient thus obtained.

Turn back to your record of Exp. 90, and compare the number of images obtained in each case with the number computed in accordance with the foregoing statement.

89. The Kaleidoscope. The fact that two plane mirrors placed at an angle of 60° to each other will form five images of an object, these images being arranged symmetrically with respect to the mirrors, has led to the construction of the *kaleidoscope*, which in its simplest form consists of two long, narrow, plane mirrors making an angle of 60° with each other. These mirrors are contained in a tube, closed at one end by a glass plate covered by a diaphragm with an aperture in its center; at the other end by a plate of ground glass, on the inner side of which lie loose fragments of colored glass. On looking through the aperture and along the axis of the tube, an observer sees a design, symmetrical about the

axis and often very beautiful, formed by the fragments of colored glass and their five reflections. Whenever the tube is shaken, the arrangement of the fragments on the ground glass is changed, and a new design appears.

EXAMPLES.

1. When the distance of a gas flame was 84^{cm} from the grease spot of a photometer, and that of a candle 40^{cm} on the other side, the grease spot disappeared. To how many candles is the gas flame equivalent?

2. An incandescent lamp equivalent to 10 candles is placed at a distance of 1^{m} from a screen. At what distance from the screen must a candle be placed in order to give the same intensity of illumination?

3. An object is placed at a distance of 4^{cm} in front of a plane mirror. How far from the object will the image appear?

4. How many images of an object will be formed by two plane mirrors, if they make an angle with each other of 18°? If they make an angle of 10°? If they are parallel?

5. The aperture of a pin-hole camera is circular in shape, that of another is triangular, while that of a third is square. What effect has the shape of the aperture upon the image formed on the screen of the camera?

6. If a candle is placed at a distance of 30^{cm} from an opaque body 12^{cm} wide, what will be the width of the shadow cast by the opaque body when the shadow falls upon a screen 100^{cm} distant from the candle?

CYLINDRICAL MIRRORS.

90. Convex and Concave Cylindrical Mirrors. The polished outside surface of a cylindrical calorimeter, such as was used in some of the experiments in heat, is a convex cylindrical mirror, while the inner surface, if polished, is a concave cylindrical mirror. The cylindrical mirrors usually met with are segments formed by cutting from a cylindrical surface strips running lengthwise of the sur-

face. The *center of curvature* of a cylindrical mirror is the center of the circle of which the portion of the mirror under consideration is the arc.

On looking into a cylindrical mirror, you will see a distorted image: when the mirror is held in a vertical position, the image has one form, but when the mirror is held in a horizontal position, the image assumes a different form. It will be the object of the next two experiments to make you acquainted with the laws of cylindrical mirrors, and so to help you to understand better the reason for the formation of the curious images which cylindrical mirrors give.

Experiment 92. *To find how the image in a convex cylindrical mirror compares with the object in position, size, and shape.*

Apparatus. A convex cylindrical mirror mounted on a semi-circular support of wood; a small sheet of paper; a small pin; a meter stick.

Directions. Lay the sheet of paper flat upon a table, and upon this sheet put the mirror (Fig. 60). With a sharp-pointed lead pencil draw a line round the support. Draw on the paper, at a distance of about 5^{cm} from the front of the mirror, a straight line about 6^{cm} long, and mark the ends and the center of this line with numbers. We shall call this line the object. Stick the pin upright into one extremity of the line. In order to get the position of the image of the pin, sight at the image along the edge of a meter stick laid on the table, and, guided by the edge of the meter stick, draw a line towards this image; then move the meter stick to a place widely dif-

ferent from the first, sight along the meter stick at the image, and towards it draw another line. Each of the lines just drawn should be numbered with the same figure as that which numbers the point into which the pin was stuck. Now stick the pin into the middle point, sight at its image, and, as before, draw lines, and number each with the same figure with which you numbered the middle point. Finally, stick the pin into the other extremity of the line, and again sight at its image, and draw lines.

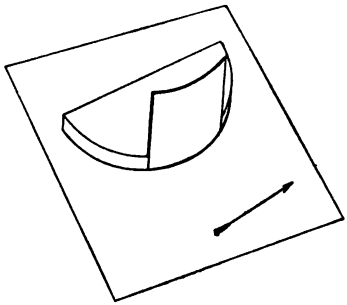


FIG. 60.

Remove the mirror from the paper, and get the position of each of the three images by producing each pair of lines till they cross. Connect by a line the three points thus found. This line is the image.

Is the image as far behind the mirror as the object is in front?

Is the size of the image larger or smaller than that of the object?

Is the shape of the image the same as that of the object?

Make a dot at the middle of the straight line forming one side of the outline of the support. This dot will mark the center of curvature of the mirror. Join by lines the center of curvature to each of the points of the line into which the pin was stuck.

Does each of these lines pass through, or nearly through, the corresponding points of the image?

Repeat the experiment on a fresh sheet of paper with a line 5^{cm} long drawn 6^{cm} from the face of the mirror.

From the results which you now obtain, do you get the same answers to the questions as before?

Did the height of the image seem to be the same as the height of the pin? Did the width of the image seem to be the same as that of the pin?

Hence, should you say that the lines of an object, which are parallel to the vertical lines of a convex cylindrical mirror, appear in the image unchanged in length, while horizontal lines in the object are shortened in the image?

91. Distance of Image from Convex Cylindrical Mirror. The law that the angle of reflection equals the

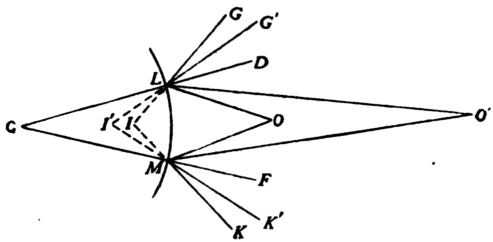


FIG. 61.

angle of incidence—a law which holds true for both curved and plane mirrors—will enable the student to see, as is indicated in Fig. 61, that the image of a point is not, as in the case of plane mirrors, so far behind a convex mirror as the point is in front, but that the image is at a less distance from the mirror than is the point itself.

The arc LM represents the mirror. Let O be the position of the object. Let OL and OM be two rays from O meeting the mirror in L and M. From C, the center of

curvature, are drawn the radii *CL* and *CM*, which are produced to *D* and *F*. The reflected rays *LG* and *MK* are drawn so as to make the angles *GLD* and *KMF* equal respectively to the angles *OLD* and *OMF*, as the angle of reflection is equal to the angle of incidence. When the lines *GL* and *KM* are produced, they meet in the point *I*, and this point gives the position of the image of the object at *O*.

In the same way the object at *O′* will have its image formed at *I′*. The image is nearer the mirror than is the object.

92. Images formed by a Concave Cylindrical Mirror. All images formed by a convex cylindrical mirror or by a plane mirror are virtual. But with a concave cylindrical mirror, both virtual images and real images can be formed. Whether the image formed by a concave cylindrical mirror will be real or virtual depends upon the distance of the object in front of the mirror, as indicated by the following construction:

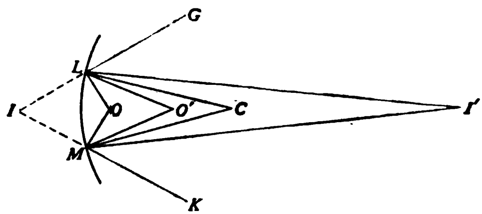


FIG. 62.

In Fig. 62 the arc *LM* represents the mirror. Let *O* be the position of the object. Let *OL* and *OM* be two rays from *O* meeting the mirror in *L* and *M*. From *C*, the center of curvature, are drawn the radii *CL* and *CM*. The reflected rays *LG* and *MK* are drawn so as to make the angles *GLC* and *KMC* equal respectively to the angles *OLC* and *OMC*, as the angle of reflection is equal to the angle

of incidence. When the lines *GL* and *KM* are produced, they meet in the point *I*, and this point gives the position of the image of the object at *O*.

If the object is at the point *O′*, the reflected rays will actually cross at *I′* in front of the mirror. It is to be noted that the point *O* is at a distance from the mirror less than half the radius of curvature, while the point *O′* is at a distance greater than half the radius of curvature. The image of an object at *O* is *virtual*, while that of an object at *O′* is *real*.

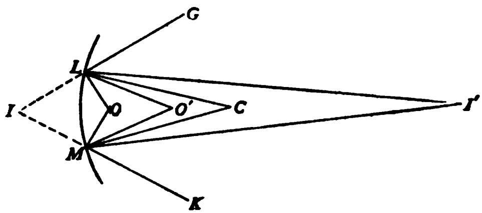


FIG. 62.

Experiment 93. *To find virtual images and real images in a concave cylindrical mirror, and to distinguish between them.*

Apparatus. The cylindrical mirror of the preceding experiment turned so as to present its concave side; two matches; a small pin.

Directions. As shown in Fig. 63, two radii should be drawn from the center of curvature to the mirror, and between these radii three arrows should be drawn, the first 1.5^{cm} from the center of curvature, the second 3.5^{cm}, and the third 4.2^{cm}.

Holding the mirror about 25^{cm} from the eye, look at the images of the three arrows.

Does each of the three images point from left to right as do the arrows themselves?

Stick a pin into the center of one of the arrows, and lay the matches, forming a wide angle with each other, so that they shall point towards the image of the pin. If the matches, provided they could be produced, are so inclined to each other as to meet behind the mirror, the image is virtual; if they are so inclined to each other as to meet in front of the mirror, the image is real. Determine in this way the nature of each of the images, whether it is real or virtual, and record the results.

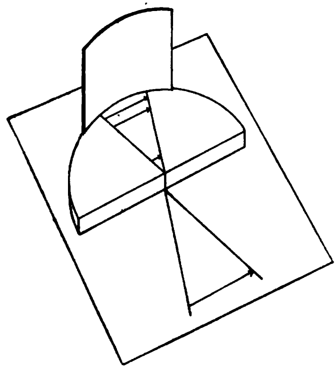


FIG. 63.

On a sheet of paper, as shown in Fig. 63, draw an arrow about 7^{cm} distant from the center of curvature.

Stick a pin into one end of this arrow, and by sighting as in the last experiment, determine the position of its image; stick the pin into the other end of the arrow and determine the position of its image; finally, stick the pin into the center of the arrow and determine the position of its image. Join the three points thus determined, and thus form the image of the arrow.

Is this a real or a virtual image?

REFRACTION.

93. Refraction of Light. The object of the next experiment will be an examination of the effect produced upon the direction of a ray of light when the ray passes from air through a piece of glass and into the air again.

Experiment 94. *To find the shape of the path followed by a ray of light in passing from an object through a prism to the eye.*

Apparatus. A prism with square ends; a sheet of paper 50^{cm} square; four tacks; four long pins.

Directions. Lay the sheet of paper on a table and fasten it in place by means of the tacks, one at each corner. On the center of the sheet stand the prism upright with one of its faces parallel to the right-hand edge of the paper, as in Fig. 64. Close to the edge of the paper farthest from you, and a little nearer the right-hand edge of the paper than the prism, stick one of the pins upright. Mark the position of this pin by the letter *A*.

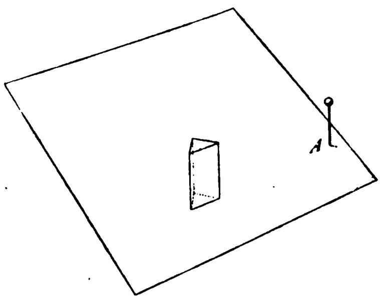


FIG. 64.

Place the eye a few centimeters above the edge of the table and look into the prism. Move the eye to the right or left, if necessary, till the image of the pin at *A* is seen through the prism. When the image is seen, stick a pin upright into the paper near the eye, so that this pin shall just hide the image when the eye is kept stationary. Mark the position of this pin by the letter *D*. Then into the paper, near the side of the prism next the eye, stick another pin so that it shall be in line with the pin at *D* and with the image. These two pins on the side of the prism next the eye give the direction in which the ray of

light moves on emerging from the prism. To get the direction of the ray of light that enters the prism from the pin at *A*, stick into the paper on that side a long pin in such a position that its image hides, when the eye is in its proper position, the image of the pin at *A*. With a fine-pointed lead-pencil draw a line round the end of the prism on the paper. Remove the prism and the pins. Lay a meter stick on the paper, and draw a line from the position occupied by the first pin, *A*, through the position of the pin next it on the same side of the prism until the line meets the line representing the position of the side of the prism. This line represents the incident ray. Then connect by a line drawn from the side of the prism the positions of the two other pins. This line represents the emergent ray. As the part of the ray of light in the prism passes straight from the point *B* where the incident ray strikes the side to the point *C*, whence the emergent ray leaves the prism, connect these two points by a straight line.

From an inspection of your diagram, how many times, and at what points, is the ray of light bent in passing from the object (the pin at *A*) through the prism to the eye?

From a point outside the prism to a point within the prism draw, through *B*, *EF* perpendicular to the side of the prism.

Definition. *A ray of light that falls upon a substance is called an incident ray* (*AB* in your diagram); *if the substance is one like glass or water which allows the ray to enter, the ray after entering is called the refracted ray* (*BC* in your diagram).

The perpendicular to the surface drawn through the point of incidence, that is, the point at which the ray strikes the surface, is called the *normal.*

The angle *ABE*, between the incident ray and that part of the normal lying in the medium (air in this case) in which is the incident ray, is called the *angle of incidence;* while the angle *FBC*, between the refracted ray and that part of the normal lying in the medium (glass in this case) in which is the refracted ray, is called the *angle of refraction.*

By inspection of your diagram, which is the smaller, the angle of incidence or the angle of refraction?

In the same manner draw at the other face of the prism, whence the emergent ray leaves, a perpendicular, producing it a little way into the prism. Letter the end of this perpendicular which lies within the prism *G*, and the end that lies without *H*.

Which is the smaller, the angle of internal incidence, *BCG*, or the angle of external refraction, *HCD?*

When a ray of light passes from a dense medium like glass into a rare one like air, is the ray bent away from the normal or towards it?

Show by a sketch that a ray of light falling obliquely upon a pane of glass has the same direction after emerging from the glass as it had before entering it.

Definition. *Refraction is the bending of a ray of light in passing from one medium into another.*

94. A Phenomenon explained. When looking into a clear pool of water perhaps you have tried to touch

some object at the bottom with a stick; unless the object lies directly beneath the eye, the stick to strike the object must not be thrust straight towards the point where the object *appears* to be, but behind it. An oar when dipped into water appears bent. A straight post standing in water also presents a very curious phenomenon. To the observer, shown on the bank in Fig. 65, the part of the post which is beneath the surface of the water appears to be shorter than it really is. This phenomenon is due

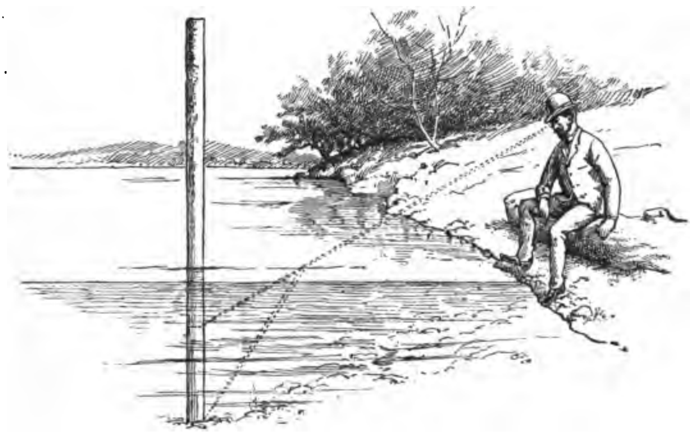


FIG. 65.

to the refraction, on emerging into the air, of the rays of light proceeding from the part of the post under the water. The eye of the observer is deceived by these refracted rays. It is unaware of the bending of the rays at the surface of the water, and so the object appears in the direction whence the rays come on leaving the water.

The apparent bending of an oar, when dipped into water, is explained in the same manner.

95. Index of Refraction. We found, when studying the subject of the *reflection* of light, that the angle of reflection was equal to the angle of incidence; but on coming to the study of the *refraction* of light, we found that the angle of refraction is not equal to the angle of incidence, being smaller than the angle of incidence if the ray passes from a rare into a dense medium, but larger if the ray passes from a dense into a rare medium.

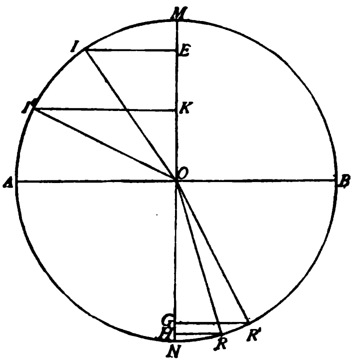


FIG. 66.

There is, however, a somewhat remote relation between the angle of incidence and the angle of refraction, which can be illustrated by the help of Fig. 66. Let AB represent the surface at which refraction takes place; IO and OR, the incident ray and the refracted ray; MN, the normal to AB at O, the point where the ray pierces the surface. With O as center and any convenient radius, describe a circumference $AMBN$. From I and R respectively draw IE and RH perpendicular to MN. Let $I'O$ represent another incident ray, and $R'O$ another refracted ray, and $I'K$ and $R'G$ the perpendiculars.

Now physicists have found by experiment that the ratio $\frac{IE}{RH}$, which is called the *index of refraction*, is always equal

to the ratio $\frac{I'K}{R'G}$, as long as no change is made in the two mediums; if one medium is air and the other glass, then the index of refraction has a uniform numerical value, no matter at what angle the incident ray enters; but if for glass we substitute water, then the index of refraction assumes a new value.

Turn back to Exp. 94, and in your diagram (which should have been very accurately constructed; if not accurately constructed, it must be drawn afresh), with the point B as a center, describe a circumference which shall cut the normal in two points, and also cut the incident ray and the part of the ray in the glass. Draw IE and RH, and with a pair of dividers carefully measure the length of each of these lines. Divide the greater length by the smaller. Record the result.

Also with C as a center repeat the process. Record the result.

FIG. 67.

Experiment 95. *To find the index of refraction from air to water, by Hall's method.*

Apparatus. A glass jar; a brass partition to fit the jar; a brass index; a meter stick; a sheet of paper.

Directions. Put the partition in place in the jar (Fig. 67), and also the brass index. Pour water into the jar till its level comes within 2^{mm} of the middle tooth of the partition. Then, looking through the side of the jar, add water cautiously till the level of the water is less than

0.5^{mm} from the tooth. With the eye about 25^{cm} from the edge of the jar, sighting in a line with the edge of the jar and the lower edge of the tooth, adjust the index till its tip seems to lie in this line produced. *After* these adjustments have been carefully made, look to see whether the water touches the tooth. In case the water has wet the lower edge of the tooth, all the adjustments must be made again.

Then measure and record the internal diameter, and also the distance of the lower edge of the tooth below the edge of the jar. Carefully measure, on the outside of the jar, the distance of the tip of the index below the edge of the jar.

Having completed the measurements, make carefully a full-sized diagram of the sides of the jar, as shown in Fig. 68. Draw the partition *GC*, making *GC* in length equal to the distance of the lower edge of the tooth below the edge of the jar. Produce *GC* by a dotted line. Through *C* draw a horizontal line to represent the surface of the water. Mark the position of *P*, the tip of the index, and draw *PC*, which represents the direction of the ray before leaving the water, and draw *CR*, representing the refracted ray. With any convenient radius, as *CR*, and with *C* as a center, describe the arc *RI*, cutting *CR* in *R*, and *PC* in *I*. From *R* drop the perpendicular *RG* on *GC*, and from *I* drop the perpendicular *IE* on *GC* produced. Measure carefully *IE* and *RG*.

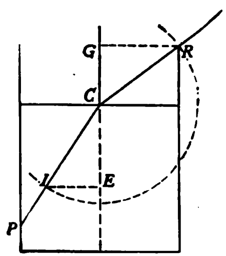


FIG. 68.

The ratio $\frac{IE}{RG}$ is the *index of refraction* from *water* to *air*, while the ratio $\frac{RG}{IE}$ is the *index of refraction* from *air* to *water*.

Compute each of these ratios.

It is a good plan to put the index on the side of the jar opposite the first position of the index, without disturbing the partition, and make a new adjustment. The average of the two results obtained from the two sets of measurements will give the index of refraction with greater accuracy, as by this means errors due to the unevenness of the edge of the jar and to the position of the partition will be made very small.

LENSES AND PRISMS.

96. Relation between Lenses and Prisms. Suppose we have a number of prisms, one of which we place at r; then a ray of light, Ar, from a spot of light, A, after

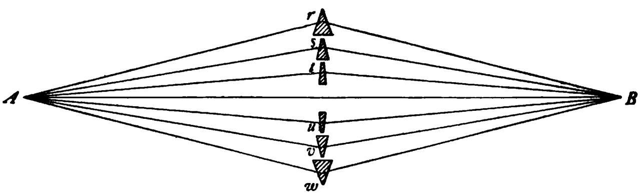


FIG. 69.

passing through the prism, is refracted. If at w a prism exactly like that at r is placed, as shown in Fig. 69, it will refract the ray, Aw. Let us denote by B the point of intersection of the two refracted rays. By using some-

what smaller prisms we can, by trial, find positions *s* and *v*, so that these prisms will refract rays from *A* to *B*. Similarly positions for another pair of prisms could be found; call these positions *t* and *u*. Now a lens may be regarded as made up of so great a number of prisms that the joinings become smooth, as shown in the diagrams in Fig. 70.

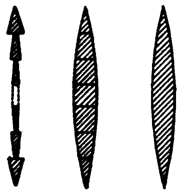


FIG. 70.

97. Definitions Relating to Lenses. Lenses are commonly made of glass, and their surfaces are usually portions of the surfaces of spheres. In the simplest case the bounding surfaces are of equal radii.

The *centers of curvature* of a lens are the points *O* and *O′*, that is, the centers of the spheres (Fig. 71) whose intersection forms the lens.

The *principal axis* of a lens is a straight line of indefi-

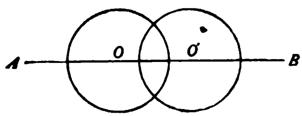


FIG. 71.

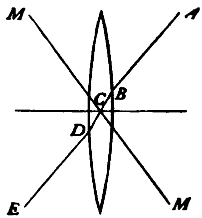


FIG. 72.

nite length drawn through the centers (*O*, *O′*) of curvature. In Fig. 71 *AB* represents the principal axis.

The *optical center* of a lens is a point, *C* in Fig. 72, so situated that a ray, *ABCDE*, passing through it has the

same direction after leaving the lens as before entering it. The position of the optical center depends upon the shape of the lens. For a lens like that represented by the figure, where the two sides are curved alike, the optical center of the lens is at the middle of the lens.

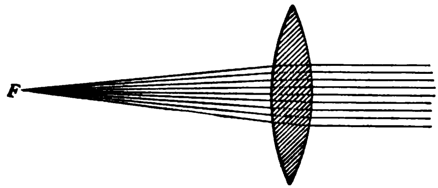


FIG. 73.

A *secondary axis* of a lens is any straight line, except the principal axis, drawn through the optical center. In Fig. 72 *MM* represents such an axis. There are an infinite number of such axes.

The *principal focus* of a convex lens is the point at which the rays, parallel before entering the lens, cross after passing through the lens.

In Fig. 73 the point *F* is the principal focus. It will be noticed that the rays before entering the lens are parallel, but that the lens bends them so that they meet after emerging.

98. Names and Properties of Different Kinds of Lenses. In the following list is given the name, and after it the description of each lens. The numbers refer to the diagrams of Fig. 74.

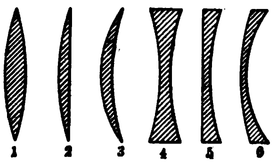


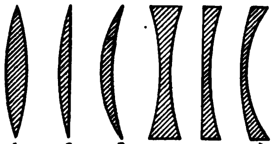
FIG. 74.

1. Double convex lens, both surfaces convex.

2. Plano-convex, one surface convex, one plane.

3. Concavo-convex, converging, one surface convex, one concave.

1, 2, and 3 are called *converging* lenses, because parallel rays of light, after passing through them, converge. Notice that they are *thickest* in the *middle*.



1 2 3 4 5 6

FIG. 74.

4. Double concave, both surfaces concave.

5. Plano-concave, one surface concave, one plane.

6. Concavo-convex, diverging, one surface concave, one convex.

4, 5, and 6 are called *diverging* lenses, because parallel rays of light, after passing through them, diverge. Notice that they are *thinnest* in the *middle*.

3 and 6 are also called *meniscus* lenses.

REAL IMAGES.

99. Focal Length; Conjugate Foci. The object of the next two experiments will be to make clear the meaning of the terms *focal length* and *conjugate foci*.

Experiment 96. *To find the focal length of a double convex lens.*

Apparatus. A spectacle lens; a meter stick; two blocks each with a groove to slide on the meter stick; a piece of cardboard 10^{cm} square; colored glasses.

Directions. Stand the piece of cardboard in the opening in the top of one of the blocks, as shown in Fig. 75, and the lens with its longest diameter vertical in the opening in the top of the other block. It is well to cement the lens in place by dipping the edge in a mixture of equal parts of melted beeswax and rosin. On the meter stick place the screen and the lens with its prin-

cipal axis parallel to the length of the rod. Point the lens toward the sun, so that the sun's rays shall pass through the lens and fall upon the screen. In order not to dazzle the eyes, look at the bright spots on the screen through colored glasses. Move the lens back and forth on the rod until a position is found that gives on the screen a sharp image of the sun. The distance from the lens to the screen as now arranged is called the *focal*

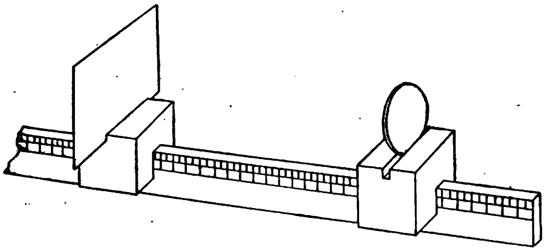


FIG. 75.

length of the lens. Record in your note-book the position of the lens on the meter stick, that is, the number of centimeters it is from one end of the rod, and also record the position of the screen, that is, the number of centimeters it is from the end of the rod from which you measured the distance of the lens. You will notice a mark carried round each block to the meter stick to assist you in making these measurements.

What will the difference of these two readings give?

Make a change in position of the screen on the meter stick, and adjust the lens till you again get a sharp image of the sun on the screen. Take the measurements and record as before. Repeat three times and record, mak-

ing five measurements of the focal length in all. Find the average of the five measurements.

The rays of light from the sun are practically parallel, and on passing through the lens are bent so as to pass through the principal focus.

Suppose a luminous point placed at the principal focus, what effect would the lens have on the rays diverging from the point?

Experiment 97. *To find the relation between conjugate focal distances of a lens and its focal length.*

Apparatus. The same as in Exp. 96, and in addition a small kerosene lamp. *Use the same lens as in Exp. 96.*

Directions. Over a smoky flame hold the lamp chimney till its outer surface is covered with soot. Light the wick of the lamp. Put the chimney in place, and with a sharp-pointed pin draw an upright arrow, as shown in Fig. 76, in the soot on the chimney opposite the bright part of the flame. *Have the lines of the arrow very fine.* Perform this experiment in a darkened room.

Place one end of the meter stick as nearly as possible directly beneath this arrow, and at the opposite extremity of this stick put the cardboard screen, so that its center shall be over the end of the stick. Support this end of the meter stick on a block. Set the lens on the stick near the lamp, and then slide it away from the lamp till a distinct image of the arrow (which we call the object) appears on the screen. Read in centimeters the distance from the center of the lens to the arrow on the chimney, and record this distance in a column headed D_o. (D stands for distance, and the letter o placed to the right and a little

below stands for object: so D_o is an abbreviation for "the distance from the lens to the object.") Measure in centimeters, and record, in a column headed D_i, the distance from the center of the lens to the image on the screen. (D_i stands for "the distance from the lens to the image.") Without moving anything but the lens, slide that along the stick till it again throws upon the screen a distinct image of the arrow. Record the distance of the lens from the lamp in the column headed D_o, and the distance of

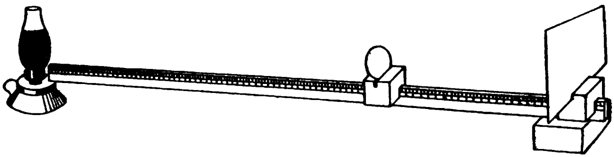


FIG. 76.

the lens from the screen in the column headed D_i. Then place the screen at a distance of 80^{cm} from the arrow on the chimney, and proceed as before.

Place the screen at distances of 70^{cm}, 65^{cm}, 60^{cm}, from the arrow on the chimney, making records in the columns headed D_o and D_i, till the lens ceases to throw a distinct image of the arrow upon the screen. In all, the screen should be put in five or six different positions. If, with the lens which you have, you are unable to get distinct images for five or six different positions of the screen, start with the screen at a distance of *more* than 100^{cm} from the arrow on the chimney, and then move the screen towards the arrow about 10^{cm} at a time.

Let F denote the focal length of the lens as already found. Express in every case the quantities $\frac{1}{D_o}$ and $\frac{1}{D_i}$ as

decimals, then add them (that is, add $\frac{1}{D_o}$ and $\frac{1}{D_i}$ expressed as decimals), and see how the sum in each case compares with $\frac{1}{F}$ expressed as a decimal.

Definition. *Conjugate foci of a lens are any two points so situated with respect to the lens that an object at either of them will produce an image at the other.*

How many conjugate foci can your lens have?

Look over the results of the experiment just performed, and answer the following questions:

If the distance between the lens and the object (the arrow) is increased, does the distance between the image and the lens increase or decrease?

When the distance from the lens to the object is large, is the image large or small?

When the lens is a long distance from the object, is the image near the principal focus?

100. Parallax. If the distance of the object from an ordinary lens is several hundred meters, the image practically coincides with the principal focus. A second way is thus suggested of finding the principal focus of a lens, and, consequently, the focal length. You have already found the principal focus of a lens in Exp. 96. If you look through a lens at a distant object such as a chimney, you will see an inverted image of the chimney. If you can locate the position of the image (a screen cannot be used for the purpose in this case), the distance from the lens to the image will be the focal length of the lens. A peculiar

means is employed to find the position of the image, which depends on the following facts:

When traveling by rail and looking from the window at the distant landscape, you have doubtless observed that objects at different distances are left behind at different rates, those not far from the train rapidly, those at a great distance slowly; consequently, they seem to move past one another. This apparent relative shift of objects due to a change in the observer's position is called *parallax.*

To illustrate the meaning of parallax still further, hold your two forefingers upright in line with each other and with the eye. Fix the eye on the nearer finger and slowly move the head from side to side by slightly bending the neck. The finger that is farther from the eye will appear to move in the same direction as the head. Now bring the more distant finger nearer the other, the relative motion between them becomes less, and finally, when the fingers are close together, there is no relative motion. It will be well to keep in mind the following:

Rule. *When two objects are in the same general direction but unequally distant from an observer, the more distant object appears to move, with respect to the nearer, in the same direction as that in which the observer's eye is moving.*

Experiment 98. *To find, by the method of parallax, the focal length of a double convex lens.*

Apparatus. The lens already used, mounted on a meter stick; a block to slide on the meter stick with a long pin stuck in a vertical position into the middle point of its upper surface.

Directions. Open a window from which you can look a long distance. Lay the meter stick on the window sill.

Put the block carrying the pin in such a position on the stick that it shall be about as far from the end of the stick as you are accustomed to hold the printed page of a book from the eye when reading. Put the lens on the meter stick (Fig. 77), *but not between the pin and the eye.* Point the stick toward some object, as a chimney several hundred feet away. When you look through the lens, you will see a little image of the chimney, but inverted. This

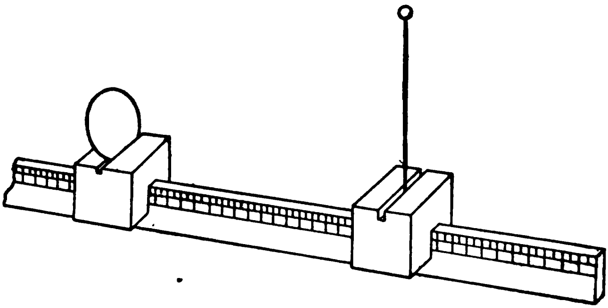


FIG. 77.

image is a real one, and exists somewhere in the space between the lens and your eye. It must be your purpose now to determine accurately the distance from the lens to this image. Move the head from side to side, all the while looking at the image. According to our rule (see page 227), if the image is farther from the eye than the pin is, it will move with respect to the pin in the same direction as you move your head; consequently, in order to get the image of the chimney to have the same position as the pin, the lens must be moved towards the pin. On the other hand, if, when the head is moved from side to side, the image moves with respect to the pin in the opposite direction to

that in which you move your head, the image is nearer the eye than the pin, and to make the image coincide with the pin, the lens must be pushed farther away from the pin. Adjust the lens till there is no relative motion between the image and the pin, that is, till the two, on moving the head, always keep accurately together.

Record the position of the lens and that of the pin.

What is the focal length of the lens in centimeters?

Repeat the setting and measurements several times. As before, record these measurements.

How does the average of the results obtained by this method compare with that obtained in Exp. 96?

Experiment 99. *To find the position of the real image of a near object, formed by a double convex lens.*

Apparatus. A meter stick; a double convex lens; three blocks with grooves to slide on the meter stick; two short pins.

Directions. Put the blocks that carry the pins on the meter stick, with the block carrying the lens between them, as shown in Fig. 78. Put one pin at a distance,

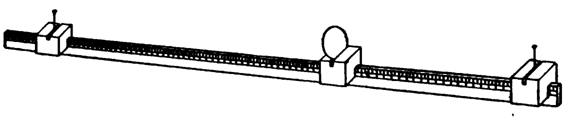


FIG. 78.

say, 1.5 times the focal length away from the lens. Slide the other pin along the meter stick, using another meter stick, if necessary, placed with its end against that of the first. By the method of parallax, find the position of the real image of the first-mentioned pin. Changing nothing,

sight from the neighborhood of the first-mentioned pin towards the image of the other.

Does this image coincide with the position of the first-mentioned pin?

Measure the distance from the lens to each of the pins.

Make another setting; measure and record the distances.

From the measurements compute the focal length of the lens.

Explain how this experiment has illustrated the meaning of the term *conjugate foci.*

Experiment 100. *To find the shape and the size of a real image, formed by a converging lens (double convex), as compared with the shape and the size of the object.*

Apparatus. The lens mounted on a block; a sheet of blank paper 100^{cm} long and 50^{cm} wide; a meter stick; two pins, one long, the other short.

Directions. Spread the paper smoothly on a table, and fasten it at the corners by means of tacks. Have the sheet laid with one end near the edge of the table. Draw a line from the middle of one end of the sheet to the middle of the other end. At a distance of 2^{cm} or 3^{cm} from that end of the sheet remote from the edge of the table, draw a line at right angles to the line first drawn. The line last drawn should be 8^{cm} long, and should extend 4^{cm} on either side of the line first drawn. Mark one end of this 8^{cm} line like the tip (Fig. 79) of an arrow. Divide the arrow thus formed into four parts of equal length by means of dots.

Lay the block carrying the lens on its side. Have the edge of the lens rest upon the paper, and the lens so placed that its axis is parallel to the median line (the line

first drawn on the paper). The center of the lens must be directly over the median line at a distance from the middle of the arrow about 1.5 times the focal length of the lens. If the lens should be placed at a distance less than its focal length from the line, a virtual image would be obtained. In order to get the center of the lens exactly over the median line, stick the short pin upright into the point 3, at the middle of the arrow. Then stick the long pin upright into the median line near the edge of the table. After you have done this, slide the lens a little way to the right or to the left across the median line, till the image of the small pin is hidden by the long pin, when the eye is held near the edge of the table and in line with the two pins. The center of the lens is now over the median line. The lens must not be moved during the subsequent parts of the experiment.

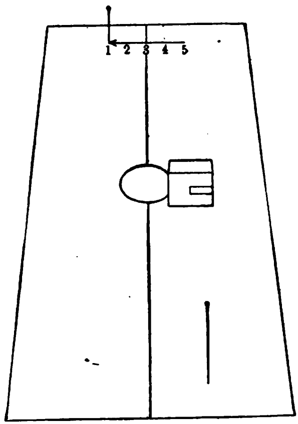


FIG. 79.

To locate the position of the image of 3, move the long pin back and forth till, by the method of parallax, you make this pin coincide with the image of the short pin. Mark this position I_3. (I stands for "image," and the small number placed to one side and a little below this letter indicates the point of the arrow into which the pin is stuck.) Now stick the short pin upright into the arrow at the tip, which we shall call point 1, and then on the other side of the lens stick the long pin upright through

the paper into the table in such a position (found by the method of parallax) as to make it coincide with the image of the pin in the tip of the arrow. Mark the position of the image, that is, the point where the second pin pierces the paper, I_1. In the same way locate the points I_2, I_4, etc., corresponding to the points 2, 4, etc., of the arrow. The student is warned in this experiment not to let any idea he may have, as to where an image-point *ought* to be, interfere with his judgment as to where the point really *is*. The five points, I_1, I_2, I_3, I_4, I_5, outline the image of the arrow as the lens would form it if the lens were lowered until its center was on a level with the paper. Remove the lens, and mark by a dot the point on the paper above which the center of the lens was. Then carefully draw, with a sharp lead pencil, straight lines joining each of the object-points (1, 2, etc.) of the arrow with its corresponding image-point (I_1, I_2, etc.). Measure the distance from each object-point to the lens (that is, to the dot marking the position of the center of the lens). Measure the distance from each of the image-points to the lens. Record all of these measurements.

Measure and record the length of the object and that of the image. If the image is not a straight line, measure the distance from I_1 *straight* to I_5, and also the actual length of the image, whatever its shape.

Measure the distance from point 3 on the object to the lens, and from the lens to the middle point of the straight line I_1I_5. Record the distances.

Call the distance from point 3 on the object to the lens, D_o. Call the distance from the middle point of the straight line I_1I_5 to the lens, D_i.

Call the length of the object, O.

Call the length of the line I_1I_5, I.

Making use of your measurements, should you say that D_o is the same part of D_i as O is of I?

Repeat your process of reasoning, using the distance from object-point 1 to lens, and from the lens to I_1, together with the values of O and I already used.

Can you, as a result of this examination, state any law for the relation between the several distances and the dimensions of the object and of the image?

Can you explain the *form* of the image?

SUGGESTIONS. The middle of the object is nearer the middle of the lens than the ends of the object are. The focal length of a lens, for rays parallel to a secondary axis, is practically equal to the focal length for rays parallel to the principal axis. Consequently, a consideration of the change made in D_i by a change in D_o, in the equation $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$, will be useful in explaining the *form* of the image.

101. Diagram to illustrate the Formation of a Real Image. The images formed by lenses thus far considered have been *real* images, that is, images that can be caught upon a screen. An inspection of the diagram (Fig. 80) will show that *real* images are formed by the actual crossing of the rays of light.

To construct the real image of an object:

Let AB denote the object, F the principal focus of the lens, O the optical center of the lens, and $A'B'$ the image.

From the point A rays of light are darting out in all directions; one of these rays must, then, be parallel to the principal axis. This ray, lettered AD, after going through the lens will pass through the principal focus, F. (Why?) Another ray, AO, will pass through the optical center, O

(see the Def. on page 220), and intersect the ray from *A*, passing through *F*, at *A′*, forming a real image of the point *A*. In like manner a real image of the point *B* is formed at *B′*. Rays of light going from other points of *AB* will form, after passing through the lens, images of the

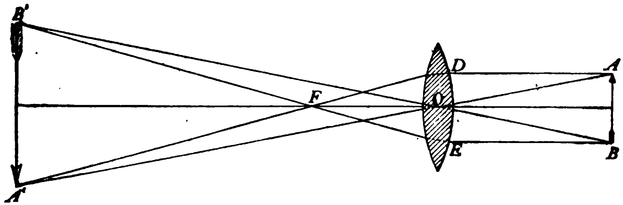


FIG. 90.

points whence they start out, so we will connect the points *A′* and *B′*, assuming according to the custom of the books on physics, that the images of all the points of *AB* will lie on the straight line *A′B′*. *A′B′* is called the image of *AB*. If the thickness of the lens is small as compared with the focal length of the lens, there is no great error in the assumption that *A′B′* is the image of *AB*. (Why?)

EXAMPLES.

1. The focal length of a double convex lens is 10^{cm}. An object is placed at a distance of 30^{cm} from the lens; at what distance from the lens will the image be formed?

SUGGESTION. Make use of the relation $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$.

2. The focal length of a double convex lens is 20^{cm}. The image of an object is formed at a distance of 100^{cm} from the lens; how far is the object from the lens?

3. The image of an object placed at a distance of 100^{cm} from a double convex lens is formed at a distance of 400^{cm} from the lens. Find the focal length of the lens.

4. Show by means of the formula $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$ what must be the distance, in terms of the focal length, F, of object and image from the lens in order that they may be of the same size.

5. At what distance from a lens of 36^{cm} focal length must an object be placed in order that the dimensions of the inverted image shall be:

(*a*) Half as large as those of the object?

(*b*) Twice as large as those of the object?

VIRTUAL IMAGES.

102. Formation of Virtual Images. The object of the two following experiments is to show the way of forming virtual images by means of a double convex lens.

Experiment 101. *To find the relation between the focal length of a lens and the distance of the object and the distance of its virtual image.*

Apparatus. A meter stick; a lens; three blocks with grooves to slide on the meter stick; a long pin; a short pin.

Directions. Mount the lens on one of the blocks, and put the block on the meter stick at a distance of about 3^{cm} from one end. Lay the meter stick with its narrow

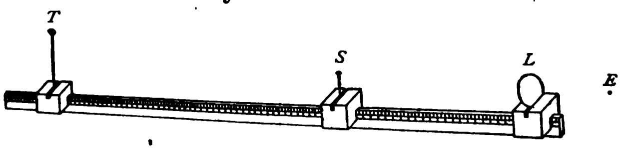


FIG. 81.

E, position of eye; *L*, position of lens; *S*, position of short pin; *T*, position of long pin.

edge on the table and with the end carrying the lens next to you. Into the center of the top of another block stick the short pin. Place this block, as shown in Fig. 81, on

the meter stick so that the pin shall be between the lens and its principal focus. The pin must, however, be on the side of the lens remote from the eye.

The long pin, stuck upright into the top of the third block, is placed on the meter stick beyond the short pin. As you look through the lens you will see a virtual image of the short pin. By moving the long pin towards you or by pushing it further away, try to locate the position of the image of the short pin by the method of parallax already explained. The eye should be held in a position which will not allow a view of any part of the short pin over the top of the lens. The long pin is to be looked at *over* the lens, not through it. When the long pin, seen over the lens, and the image of the short pin keep together as the head is moved from side to side, measure and record *the distance from the lens to the short pin*, and also *the distance from the lens to the long pin* (the position of the virtual image). Then make a new setting of the short pin, and by adjusting the position of the long pin, again find the position of the image. As before, record the readings.

In Exp. 97 you found, by substituting the proper quantities, that $\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$, that is, the reciprocals of the object-distance and image-distance *added* produce the reciprocal of the focal distance.

In the present experiment the object-distance has always been less than the focal distance, but the image-distance always greater than the focal distance.

Take the *difference* between the reciprocal of the object-distance and the reciprocal of the image-distance, that is,

$\frac{1}{D_o} - \frac{1}{D_i}$, and see how this difference compares with the reciprocal, $\frac{1}{F}$, of the focal distance.

The difficulty often met with in getting the position of the virtual image in the experiment just performed introduces an uncertainty into the value of the focal length calculated from the data obtained.

Experiment 102. *To find the shape and the size of a virtual image, formed by a converging lens (double convex), as compared with the shape and the size of the object.*

Apparatus. The same as that used in Exp. 100 with a fresh sheet of paper.

Directions. Fasten the paper on the table in the same position as for Exp. 100. Draw a median line lengthwise of the paper. Place the lens on its side, as in Exp. 100, but at a distance of about 20^{cm} from the end of the paper *nearest* the edge of the table. On the median line, on the side of the lens remote from the edge of the table, make a dot at a distance from the lens equal to about two-thirds its focal length. Through this dot draw a line at right angles to the median line. This line should be 5^{cm} long and should be bisected by the median line. Mark one end of this line like the tip of an arrow. Divide the arrow into five equal parts, as indicated in Fig. 82.

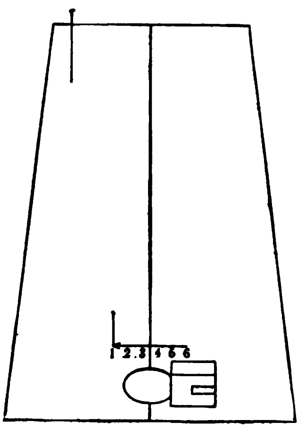


FIG. 82.

In order to get the center of the lens over the median line, stick the short pin upright into the middle of the arrow, at the point where it crosses the median line. Stick the long pin upright into the median line, at the end of the paper remote from the lens. Look through the lens with the eye in line with the two pins, and slide the lens a little way to the right or to the left, across the median line, till the image of the short pin is in line with the two pins. When the image is in line with the two pins, the center of the lens is over the median line.

Stick the short pin upright through point 1. Look through the lens and locate, by means of the long pin, the position of the image of the short pin. As this image is virtual, it will appear on the same side of the lens as is the object.

In this manner locate the position of the image of the short pin for each of the remaining five points. Draw lines and make measurements similar to those of Exp. 100.

Is there any distinction between the *form* of the image obtained in this experiment and the *form* of the image obtained in Exp. 100?

What relations can you make out, from your measurements, between the distance of object and image from the lens and their lengths?

103. Diagram to illustrate the Formation of a Virtual Image. By an inspection of the diagram (Fig. 83) it will be seen that a *virtual* image is formed not by the *actual* crossing of the rays of light, but by their *apparent* crossing.

To construct the virtual image of an object:

Let *AB* denote the object, *F* the principal focus of the lens, *C* the optical center of the lens, and *A′B′* the image which we wish to find. From *A* rays of light are darting out in all directions; one of these rays must, then, be parallel to the principal axis. This ray, lettered *AD*, after going through the lens will pass through the principal focus, *F*. (Why?) Another ray, *AC*, will pass through the optical center, *C*. (See the Def. on page 220.) The bending of the ray, *ADKF*, deceives the eye, and the point

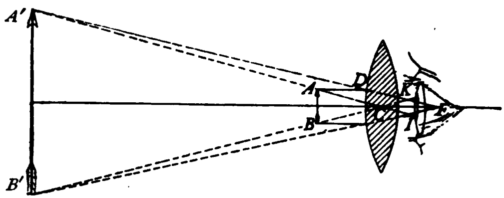


FIG. 83.

A appears to lie on the prolongation of *FK*; the point *A* also lies on the line *IC*, hence the point *A* *appears* to lie at the intersection of *FK* produced and *IC* produced, or at *A′*. *A′* is the virtual image of *A*. The point *A′* is not the point from which the rays of light actually come, but it is the point from which to the eye they appear to come.

By a similar process *B′* is found to be the virtual image of *B*.

The image, *A′B′*, is larger than the object, *AB*. Whenever an object is placed *between* a convex lens and its principal focus, a virtual image is seen on looking in from the other side of the lens. As the image is always mag-

nified, or enlarged, the name *magnifying glass* is commonly given to a lens used in this way.

In order that the straight line $A'B'$ may represent the image, with very small error, must the thickness of the lens be large or small as compared with the focal length of the lens?

104. Velocity of Light. There are four moons which revolve about the planet Jupiter. At times a moon will be visible, at others eclipsed. The exact time can be calculated at which an eclipse can be seen from the earth, when the earth is in that part of its orbit nearest Jupiter; but as the earth recedes from Jupiter, the eclipses do not occur on time, but occur later and later till the earth reaches that part of its orbit most distant from Jupiter, when the eclipses are 16 minutes and 36 seconds behind time. As the earth sweeps round in its orbit and approaches Jupiter, the eclipses occur more and more nearly on time, and when the earth and Jupiter are nearest each other, the eclipses are once more precisely on time. This discrepancy between the computed time at which the eclipse should take place and the actual time at which the eclipse is seen, when the earth is most remote from Jupiter, is due to the fact that the light has taken 16 minutes and 36 seconds in crossing the earth's orbit, a distance of about 190,000,000 miles. This gives, by dividing 190,000,000 by 996 (the number of seconds in 16 minutes and 36 seconds), the velocity of light to be about 190,000 miles per second.

105. Nature of Light. All men of science are agreed that a ray of light represents a motion of some kind.

Whenever a stone is thrown from the hand, or an arrow shot from a bow, we know that either of them will be propelled through the air with a velocity which depends upon the weight and the shape of the object, and also upon the force employed in its discharge. When we hear that light has a velocity of about 190,000 miles per second, we feel a keen interest to know what is the nature of the motion that is propagated through a distance so great in so short a period of time.

Two hypotheses have been advanced for explaining the nature of light, the *corpuscular hypothesis* and the *wave hypothesis.*

The *corpuscular hypothesis*, long supported by the authority of Sir Isaac Newton, supposes, in brief, that particles, called corpuscles, so very minute that they cannot be weighed are given out bodily, like sparks, from the sun, the fixed stars, and all luminous bodies. It is further supposed that these particles travel with enormous velocity, and excite the sensation of vision by striking against the eye.

As this hypothesis failed to explain all the facts known about light, the hypothesis was abandoned early in the nineteenth century in favor of the *wave hypothesis*, which has not only explained all the facts about light with which we are acquainted, but has also made predictions about light which have subsequently been verified by experiment. The wave hypothesis supposes the existence of a substance, called *the ether*, which occupies all space, not only the regions between the stars, but also the spaces between the molecules of all bodies. This hypothesis also supposes that a luminous body has the power to cause

waves in the ether, which travel with great rapidity, and on striking the eye produce the sensation which we call light.

When we studied the subject of wave-motion, we found that it was possible for two water waves of equal size to interfere in such a way as to produce a *calm*. In the subject of sound we found that two equal sound waves could interfere to produce *silence*. In light, two light waves can interfere and produce *darkness*, as can be readily seen by looking at a flame through a narrow slit cut by a penknife in a card. Very narrow black bands will be seen running parallel to the slit on each side of it. These black bands are formed by the interference of the light which is reflected from the edges of the slit. These edges correspond to the two centers of disturbance that we considered when examining the interference of two sets of water waves.

As an example of one of the remarkable predictions of the wave hypothesis of light, perhaps the following will serve as well as any. By a mere manipulation of the mathematical symbols by which the wave hypothesis is expressed, it was shown that by stopping, in a certain way, a portion of the rays of light passing through a circular opening, the illumination of a point upon a screen behind the opening would be greatly increased. On performing the experiment which the mathematical formulae suggested, the prediction about the increase of illumination was verified.

We must bear in mind, however, that, although the hypothesis of the existence of the ether has enabled us to explain the known facts about light, it does not by any means follow that the ether has an actual existence, not

even if facts have been predicted by its aid. For whenever a mass of facts, like those connected with the phenomena of light, is collected, and a means found to bind the facts together and to explain them, it cannot be at all surprising, if certain facts, of which we were ignorant, should be included in the collection. These facts of which we were ignorant will be found when we carefully examine the collection.

EXAMPLES.

1. In determining the illuminating power of a gas flame by Bunsen's photometer, the distance from the gas flame to the grease spot was 90^{cm}, and from the grease spot to the standard candle 35^{cm}. What was the candle power of the gas flame?

2. If a plane mirror recedes from a fixed object at the rate of 10 ft. a second, at what rate will the image recede from the mirror? From the object?

3. What must be the length of a plane mirror in order that an observer may see his whole length therein, the mirror being placed parallel to the observer?

4. When a real image is thrown upon a screen it can be seen from all points from which the face of the screen can be seen. When the image does not fall upon a screen, the region from which it can be seen is much more restricted. Explain this difference by means of a diagram showing the course of the light rays.

5. The image of a clock face is thrown upon a screen. The time is 12.30. Make a diagram of the *image* as seen by an observer looking from the lens.

6. In a photographic camera, using a single lens, let the plate be so placed that the center only of the picture is distinct. Must the plate be pushed nearer the lens or pulled further away in order that the edges of the picture may become distinct?

7. An object is placed at a distance of 8^{cm} from a lens, the focal length of which is 24^{cm}; will the image be real or virtual? Erect or inverted? At what distance will the image be from the lens?

8. An object 7^{cm} high is placed at a distance of 50^{cm} in front of a lens; the image is 1^{cm} high; what is the focal length of the lens?

9. At what distance from a lens must an object be placed so that the image shall be erect and twice as high as the object?

10. An object placed 5^{cm} before a lens has its image formed 15^{cm} from the lens on the same side; what is the focal length of the lens?

11. An object placed 5^{cm} before a lens has its image erect and of three times its linear magnitude; what is the focal length of the lens?

12. A candle and a gas flame are placed 180^{cm} apart. If the gas flame is equivalent to four candles, where must a screen be placed on the line joining the candle and the gas flame, in order that it may be equally illuminated by each of them?

13. Two parallel plane mirrors, *A* and *B*, face each other at a distance of 5 ft., and a small object is placed between them at a distance of 2 ft. from *A*, and, consequently, 3 ft. from *B*. Calculate the distances from *A* of the two nearest images that are seen in *A*, and also calculate the distances from *A* of the two nearest images that are seen in *B*.

14. Two plane mirrors, resting in a vertical position upon a horizontal table, make an angle, *A*, with each other. A ray of light from the point *P* strikes one of the mirrors at the point *B*, whence the ray is reflected to the other mirror which it strikes at the point *C*, and is then reflected in the direction *CQ*. Prove that the angle made by *CQ* and *PB* is two times the angle *A*.

CHAPTER VI.

MECHANICS.

106. Mechanics defined. Mechanics is that branch of physics which deals with the effects of force upon matter. Whenever several forces act upon a body at rest, one of two things happens, either the body remains at rest, or else the body moves. For convenience, the subject of mechanics is divided into two parts, according as the forces produce rest or motion. All cases of forces producing rest are grouped under the head of *statics*, while all cases where motion is produced are grouped under the head of *kinetics*. In the greater part of this chapter, experiments in statics only will be considered.

107. Mass; Unit of Force; Weight. A *force* has already been defined on page 141 as a *push or a pull*. Before defining the *unit of force*, which we shall use in a good deal of our experimental work, it will be well to discuss the meaning of the term *mass*, a term which we shall find useful in defining the unit of force.

The *mass* of a body is usually defined as the *quantity of matter* the body contains. To illustrate this definition of mass, suppose two pieces of iron of equal size and alike in every particular be placed together; then the mass of the two combined will be twice the mass of either. If three pieces of iron of the same size and alike in every particular be placed together, the mass of the three combined will be three times the mass of any one.

The study of physics has already brought before us the necessity of having units to measure various quantities, as length, volume, and heat, so it will not seem strange to look about for some unit of mass.

There is a piece of platinum which is the standard for mass. Copies of this standard have been made. The name of this unit of mass is the *pound*. In a grocery shop you may see lying on the counter pieces of iron with numbers upon them. These pieces of iron are the pounds and multiples and fractions of the pound. They have been made by placing one of the standard units of mass in one pan of a balance and a piece of iron in the other pan, the piece of iron being filed till it just balances the standard unit. The piece of iron is then said to be of the same mass, that is, it contains the same amount of matter as the standard. The grocer in selling his goods puts one of the pieces of iron, the pound, for example, into one pan of the balance; into the other, the article to be sold. When a sufficient amount of the article, tea, for instance, is put into the pan, the unit of mass, the pound, is balanced by the tea. Having stated what the unit of mass is, the *unit of force* is given by the following:

Definition. *The unit of force, the pound, is the pull of the earth on the unit of mass, the pound.*

It is unfortunate that the unit of force has the same name as the unit of mass.

We shall measure forces by means of the spring balance, an instrument made of a spiral spring fastened at one end to a support, the inner side of the frame; at the other end of the spring is a little index, or pointer, which moves

in front of a scale. If it were not already constructed, we might make the scale of the balance in the following way: Upon the hook of the balance hang a mass of 1 lb. Mark the place at which the pointer comes to rest. Then hang on 2 lbs., and mark the position of the pointer. Proceed in this way till the scale is completed. We could, if we wished, subdivide the pound divisions into halves and quarters, and thus obtain the fraction of a pound. Would this method of making the subdivisions be accurate? (See Hook's Law, page 152.)

Definition. *The weight of a body is a force, the earth's pull upon the body.*

NOTE. The word "weight" as commonly used is ambiguous. We speak of a certain *weight* of tea. If the tea has been weighed with a spring balance, the use of the word "weight" in the preceding sentence is correct. (Why?) On the other hand, if a beam balance (platform balance) had been used, the use of the word "weight" in the sentence is, strictly speaking, incorrect. (Why?)

If we could restrict the word "weighing" to the operation performed with the spring balance, and could use the word "massing" to mean the operation performed with the beam balance, what would be the advantage?

REPRESENTATION OF FORCES.

108. How Forces are represented. It has been found convenient to represent forces by straight lines. Thus if a force of 5 lbs. is made to act in a northerly direction upon a body, mathematicians represent this force by a straight line drawn toward the north. Then to show that the line represents a force of 5 lbs., the line is made five times the length of a line representing a force of 1 lb. (The length of the line representing 1 lb. is chosen at pleasure.)

109. Illustration of how a Line may be used to represent a Force. Let us suppose a line 0.5^{cm} long to represent in magnitude a force of 1 lb. The irregular outline of Fig. 84 representing the body, the line *AB*, 2.5^{cm} in length, drawn toward the top of the page represents both the direction of the force and also its magnitude, or size. It is convenient to indicate the direction in which a force acts by an arrowhead, as in the figure. The point *B* of the body at which the force is applied is called *the point of application of the force.* The line of indefinite length, of which *AB* is a limited portion, is called *the line of action of the force.* In order to represent completely a force on paper, three things must be known:

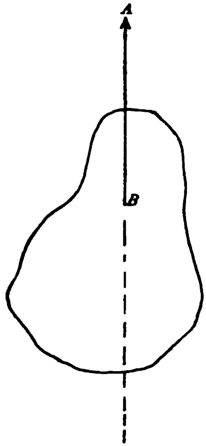


FIG. 84.

(1) The magnitude of the force;

(2) The direction of the force;

(3) The point of application.

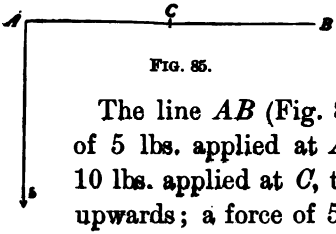


FIG. 85.

The line *AB* (Fig. 85) represents a rod. A force of 5 lbs. applied at *A* acts downwards; a force of 10 lbs. applied at *C*, the middle point of *AB*, acts upwards; a force of 5 lbs. applied at *B* acts downwards.

In representing the force at *A*, what unit of length has been taken to represent the magnitude of 1 lb.?

HINT. Measure the line with a meter stick.

Using the same unit of length to represent the magnitude of a force of 1 lb. as was used in representing the same force at A, draw lines to represent the force acting at C and the force acting at B.

EQUILIBRIUM.

110. Conditions of Equilibrium. Two or more forces are said to be in equilibrium, or to balance, when they are so opposed to each other that their combined action on a body produces no change in its rest or motion.

To determine what relation must exist among a set of forces in order that the forces may neutralize each other in their action on a body to which they are applied will be our immediate task. These conditions are called the *conditions necessary for equilibrium*, or, more briefly, *conditions of equilibrium.*

The purpose of the next experiment is to find the conditions of equilibrium of three parallel forces whose lines of action all lie in one plane, and whose points of application lie in the same straight line which is at right angles to the line of action of the forces.

In trying to discover and state these conditions, the student must keep in mind: (1) the magnitudes of the forces under consideration, (2) their directions, (3) their points of application.

There are various forms in which the conditions of equilibrium may be stated; one of these forms, good, although not the most concise, consists of answers to the following questions:

(1) How does the *magnitude* of the largest force compare with the sum of the *magnitudes* of the other two forces?

(2) How does the *direction* of the largest force compare with that of the other two forces?

(3) What is the position of the *point of application* of the largest force with respect to the position of the points of application of the other two forces?

Experiment 103. *To find the conditions of equilibrium of three parallel forces which act in one plane, and whose points of application all lie in the same straight line.*

Apparatus. Three 30-pound spring balances; a board 1 ft. square protected from warping by pieces fastened to the edges; the top of this board is divided into squares each 2 in. on a side; at the corner of each is a hole, and the holes are numbered (Fig. 86) from 1 to 49 inclusive; several iron pegs to fit the holes rather closely; three marbles equal in size; three wooden screws with attachments to fit the edge of the table; supports for the balances; a large sheet of common window glass.

Directions. On the top of a table lay a sheet of window glass, in order to have a smooth surface, and put the marbles on the glass at equal distances from one another, so that they shall be at the vertices of an equilateral triangle. Put a peg into hole 27, one into hole 25, and one into hole 23. Lay the board, marked side up, on the marbles. At one end of the table put two of the wooden screws, as shown in Fig. 87, fitting them to the table by slipping the slot, cut in the piece of wood attached to the screw, over the edge of the table. When attached to

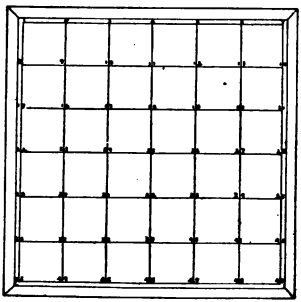


FIG. 86.

the table, the centers of these pieces through which the screws pass should be as far apart as the holes (23, 27) in the board are distant from each other. At the other end of the table, and directly opposite the portion of the table midway between the two wooden screws already in position, place the remaining wooden screw. Find and

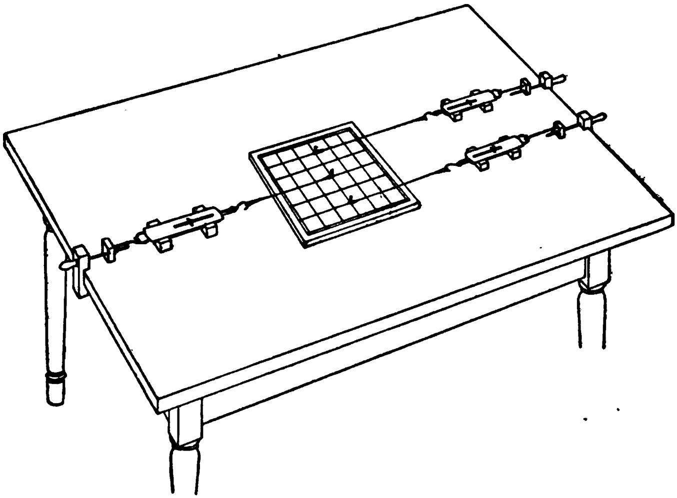


FIG. 87.

record the correction of each balance (see Exp. 58). Place the ring of a spring balance over the hook that projects horizontally from the top of the little nut on the screw. Lay the spring balance on its back, and support the balance frame near each end by little pieces of wood, as shown in the figure. In like manner fasten the other two balances to the other hooks, and support them in the little rests. Have the nuts near the ends of the screws. By means of strings, not doubled, but with loops in their ends, attach

the hooks of the spring balances to the pegs in such a way that the strings shall be parallel to each other. The hooks of the balances must not touch the board; the strings must not press down on the edge of the board; but the end of each string which passes over the peg must rest on the board. There must be no friction in any of the balances. (Why?) By turning the screws, the strings attached to the balances can be brought directly over the lines traced on the board at right angles to the line in which the pegs stand. In order to effect this, it may be found necessary to move one of the screws along the edge of the table for a little distance.

When this adjustment has been completed, turn the screws, keeping the strings over the proper lines, until the balance attached to hole 25 registers 14 lbs., when the correction for the error due to its horizontal position is taken into account. In all work involving the use of one or more spring balances in the horizontal position, a correction must be applied to the reading of *each* spring balance.

A record should be made in the note-book of the magnitude of each force, its direction, and its point of application. For ready reference, a diagram of the board should be made in the note-book, and the points indicating the holes should be numbered from 1 to 49, like the holes on the board. A brief and clear record can be made as indicated below:

HOLE.	FORCE.	DIRECTION.
23, 25, 27,	— 14, —	↓ ↑ ↓
— — —	— — —	— — —
— — —	— — —	— — —

The space on each side of the 14 should be filled with the numbers representing the magnitudes of the forces whose points of application are at 23 and 27. The first arrow indicates the direction of the force applied at 23; the second arrow that of the force at 25; and the third arrow that of the force at 27.

Next, having slackened the balances, take the peg out of hole 27 and put it into hole 26, and move the screw along the edge of the table till it is opposite hole 26. Proceed as before, but apply a force of 15 lbs. to the peg at hole number 25. Record as before.

Slacken the spring balances. Take the peg out of hole 23 and put it into hole 22. Move the screw along the edge of the table till it is opposite hole 22. Apply 16 lbs. to the peg in hole 25. Record as before.

Answer the following questions:

In each case, how does the magnitude of the largest force compare with the sum of the other two forces?

What is the direction of the largest force as compared with the direction of the other two forces?

Is the point of application of the largest force always between the points of application of the other two forces?

Divide the greater outside force by the smaller; also the greater distance from the middle force by the smaller distance. (The "middle force" is the force whose point of application is anywhere between the two outside forces.)

State the relation between the two outside forces and their distances from the middle force.

Divide any one of the three forces in one of the groups by either of the other forces. Divide the distance of this last chosen force, from the force not chosen, by the

distance of the first chosen force, from the force not chosen.

State the relation between any two forces and their respective distances from the remaining force.

State in as few words as possible the three conditions of equilibrium which must hold in order that any group of three parallel forces may balance.

RESULTANT AND EQUILIBRANT.

111. Resultant and Equilibrant defined. The *resultant* of two or more forces is a single force that will exactly replace them in its action on a body.

Thus, in Exp. 103, the resultant of the two outside forces in any case would be a force which has the same direction as the two outside forces, which is equal to their sum, and which is applied at the peg in hole 25.

The resultant of one of the outside forces and the middle force would be a force equal to their difference, applied at the peg to which the other outside force is applied, but in the opposite direction, that is, in the direction of the greater force.

The *equilibrant* of a set of forces is a single force that will exactly neutralize their action.

Thus, in any one of the cases of Exp. 103, the middle force is the equilibrant of the two outside forces. Either outside force is the equilibrant of the middle force and the other outside force.

To find the resultant of a group of any number of parallel forces, we must replace two of the forces by their resultant, then replace the resultant just found and one of the remaining forces by their resultant, and

proceed in this manner till the resultant of the group is obtained.

The special name of *couple* is given to a group consisting of two parallel forces, equal in magnitude, but acting in opposite directions. The perpendicular distance between the lines of action of the couple is called the *arm* of the couple. The effect of a couple is to rotate the body to which it is applied.

EXAMPLES.

1. A force of 6 lbs. acts due north and a force of 15 lbs. acts due south. If both forces have the same point of application, find the direction and magnitude of their equilibrant; of their resultant.

2. A rod extends east and west. A force of 10 lbs. and a force of 5 lbs., both acting due south, are applied to the rod at points 6 ft. apart. Find the equilibrant of the two forces and also their resultant, stating the direction, magnitude, and point of application of the equilibrant and of the resultant.

Solution. From the conditions of equilibrium which we have found, it follows that the direction of the equilibrant must be due north. (Why ?) It also follows that the magnitude of the equilibrant must be $10 + 5 = 15$ lbs. (Why ?) To find the point of application of the equilibrant, a little more work is necessary. From the remaining conditions of equilibrium, if C (Fig. 88) is the point of application of the equilibrant, we have

$$10 : 5 = CB : CA.$$

But $AB = 6$ ft.

Consequently $CB = 6 - CA$;

hence $10 : 5 = 6 - CA : CA.$

$$10\,CA = 30 - 5\,CA.$$

$$15\,CA = 30.$$

$$\therefore CA = 2.$$

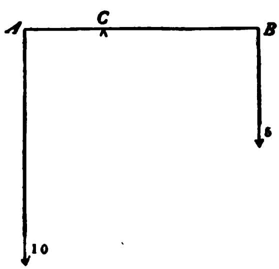


FIG. 88.

Hence $CA = 2$ ft., that is, the point of application of the equilibrant is 2 ft. from the point at which the force of 10 lbs. is applied.

The point of application of the resultant, as well as its magnitude, is the same as for the equilibrant, but its direction is due south. (Why ?)

3. A rod extends east and west. To this rod a force of 15 lbs. is applied due north and 5 lbs. due south. If the points of application are 9 ft. apart, find the direction, magnitude, and point of application of their equilibrant; of their resultant.

4. To a rod extending east and west are applied four forces: a force of 5^g due north, a force of 10^g due south, a force of 15^g due north, and a force of 20^g due south, at distances of 10^{cm}, 20^{cm}, 30^{cm}, and 40^{cm}, respectively, from the western end of the rod. Find the direction, magnitude, and point of application of their equilibrant; of their resultant.

5. A rod extending east and west is acted upon by a force of 20^g due north and by a force of 20^g due south. If the points of application are 90^{cm} apart, find their equilibrant.

Solution. The two forces given in the example form a *couple.* There is no single force that will balance a couple. This statement may be better understood by the student after examining the following investigation:

If the force acting toward the south is a little less than 20^g, the equilibrant will be a small force acting toward the south, but having its point of application a long distance from the middle force. (Why?) As the force acting due south approaches 20^g as its limit, the equilibrant approaches zero as its limit, and its point of application moves farther and farther away along the line. Hence, mathematicians say that the equilibrant of a couple may be regarded as a zero force acting at an infinite distance.

6. Find the directions, magnitudes, and points of application of two forces that will just neutralize the tendency of the pair of forces given in Example 5 to rotate the rod.

EQUILIBRIUM.

112. Points of Application not all in the Same Straight Line. For each case of equilibrium that we have tried, the points of application of the forces have been in a straight line, which was perpendicular to the direction of the forces. In Exp. 104 we shall examine cases in which the points of application of the forces are not all in the same straight line.

Experiment 104. *To find, provided three given parallel forces are in equilibrium when their points of application lie in the same straight line, whether the forces will still be in equilibrium when their points of application lie in a broken line.*

Apparatus. The same as in Exp. 103.

Directions. Set up the apparatus, using one of the cases of equilibrium already recorded in Exp. 103. Then keeping two points of application unchanged, vary the place of the third with a view of finding any positions it may have such that when the forces are just the same in magnitude and direction as at first, they shall still be in equilibrium. Experiment with each point of application in turn, and, using the form suggested in Exp. 103, record the result of the trials in your note-book.

From an inspection of your record, what inference can you draw?

SUGGESTION. If points are found at which the forces may be applied and the equilibrium holds, consider how these points are situated with respect to each other and with respect to the line in which they at first had their places.

Experiment 105. *To find, provided three given parallel forces are in equilibrium, whether they will remain in equilibrium when they are kept parallel, but veered round in a new direction.*

The question we wish to settle may be made a little clearer by an inspection of the diagrams on the following page.

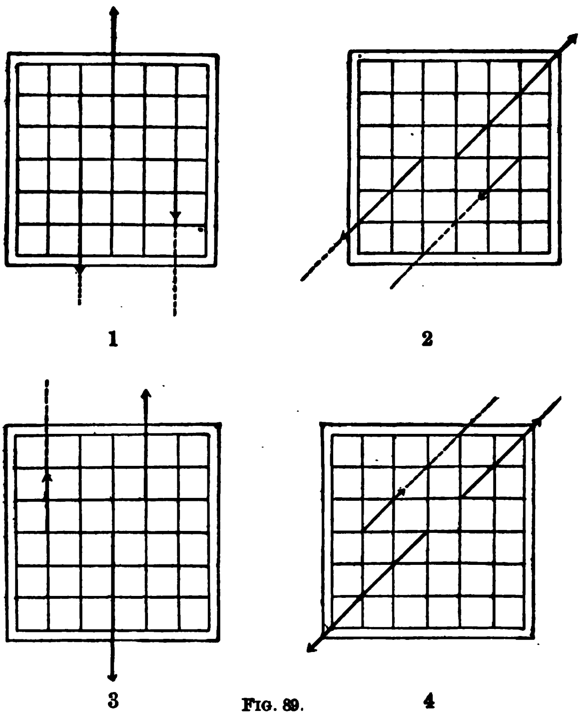


FIG. 89.

We wish to find, provided the forces in 1 and 3 (Fig. 89) are in equilibrium, whether they will remain in equilibrium when veered round as in 2 and 4.

Apparatus. The same as in Exp. 103.

Directions. Set up the apparatus, making use of one of the cases of equilibrium found in Exp. 103. Then, keeping the magnitude and point of application of each force unchanged, veer the forces round, with respect to the board, into a new direction, but keep them parallel to

each other. The easiest way to veer them round is to tighten one of the screws, and to loosen the others till the strings, remaining parallel to each other, no longer lie along the lines traced upon the board. Record the trials and the results in your note-book.

What conclusion do you draw from the experiment as thus far performed?

Repeat the experiment, using any case of equilibrium to begin with found in Exp. 104, where the points of application do not lie in the same straight line. Record the trials and the results.

Do you find the same result as in the first part of the experiment?

What general conclusion can you draw from this experiment?

MOMENT OF A FORCE.

113. Definition of the Moment of a Force. Before discussing the record of the results of Exp. 105, we will define the meaning of the term *moment of a force.*

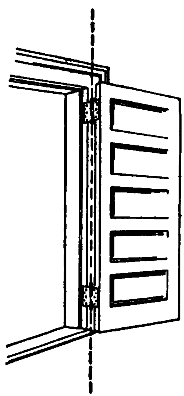


FIG. 90.

The moment of a force is the power the force has to rotate the body, to which it is applied, about some selected axis. This power depends not only upon the magnitude of the force, but upon its position.

A force applied to the middle of a door has less power to turn the door on its hinges than if it were applied to the side of the door remote from the hinges. The "selected axis" in this case is the line

running through the hinges about which the door turns, as shown in Fig. 90.

Definition. *The numerical measure of the moment of a force, with respect to an axis, is the product of the force and the perpendicular let fall on its line of action from the axis.*

Let the irregular outline represent a body; let a force, *F*, act on the body in the plane of the paper; let an axis, perpendicular to the paper, pierce it at *O*. Then the moment of the force, *F*, about the axis through the body at *O* is $F \times OM$, where *OM* is the length of the perpendicular let fall from *O* upon the line of action of *F*.

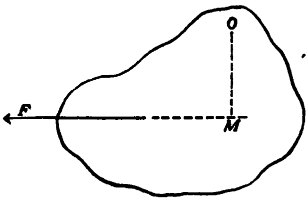


FIG. 91.

Now work out in your note-book the moments for two cases of equilibrium in Exp. 103, for two cases in Exp. 104, and for two cases (if you found as many) in Exp. 105. In each case take in turn for the axis each of the three pegs used in that case, and at least one other peg. This will make twelve moments calculated for each case of equilibrium. The calculations, however, are extremely simple. Call a moment *positive* if it *tends* to produce rotation round the given axis in the *direction of the motion of the hands of a watch.* Call a moment *negative* if it *tends* to produce rotation in the *opposite direction round the axis.*

Is there anything in the nature of the case why we should regard one direction of rotation positive rather than the other?

We can now replace the set of three conditions, for the equilibrium of parallel forces, suggested on pages 249 and 250 by a set of two conditions only:

(1) The algebraic sum of the forces must be zero; that is, the sum of the forces in one direction must be equal to the sum of the forces in the opposite direction.

(2) The algebraic sum of the moments of the forces must be zero; that is, the tendency for rotation in one direction must be equal to the tendency in the opposite direction.

NOTE. The algebraic sum of two or more quantities is the result obtained by adding them according to the rules of algebra, which take into account the signs of the quantities.

The set of conditions given above is true for the equilibrium of any number of parallel forces in one plane. To illustrate the applications of the conditions of equilibrium just stated, let us find the equilibrant of the set of forces described in Fig. 92, and consequently the resultant which always has the same magnitude as the equilibrant, but the opposite direction.

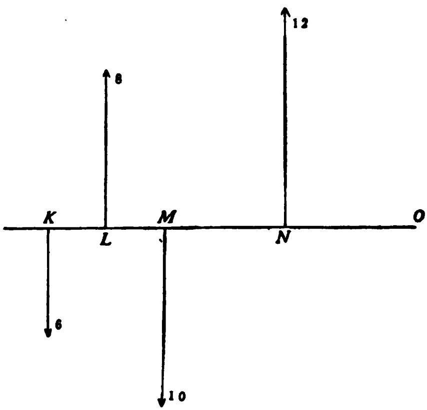


FIG. 92.

We will call forces acting upwards positive, and those acting downwards negative.

The distance $KL = LM = 2$, and $MN = NO = 4$.

Making use of condition (1), that is, in order to have equilibrium, the algebraic sum of the forces acting on a body must be zero, we find, x being the magnitude of the equilibrant,

$$x - 6 + 8 - 10 + 12 = 0, \text{ or } x = -4.$$

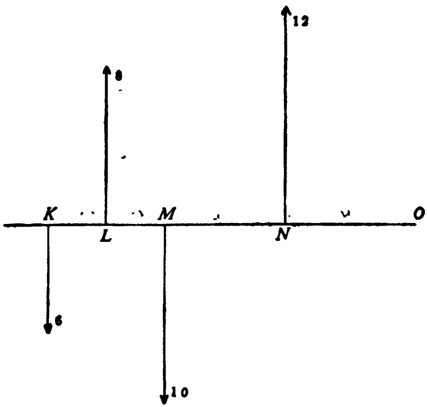


FIG. 92.

The magnitude of the equilibrant is then 4. The minus sign denotes that the equilibrant acts downwards. (Why?)

To make use of condition (2), let us first calculate the moments with respect to an axis passing through K.

Moment of	6	about	K	$=$	$(6 \times 0) = 0.$
"	" 8	"	"	$=$	$-(8 \times 2) = -16.$
"	" 10	"	"	$=$	$(10 \times 4) = 40.$
"	" 12	"	"	$=$	$-(12 \times 8) = -96.$

Moment of the equilibrant, 4, at the unknown distance y from $K = 4 \times y = 4y$.

Hence, $4y + 0 - 16 + 40 - 96 = 0$, or $4y = 72$.

Hence, $y = 18$ units to the right from K. Why is not the distance 18 units to the *left* from K?

We see that the first condition gives us the direction

and magnitude of the equilibrant; the second condition, the point of application of the equilibrant.

Let us now take moments about *M*, that is, apply the second condition with respect to an axis through *M*.

Moment of	6	about	$M =$	$-(6 \times 4) =$	$-24.$
"	" 8	"	" $=$	$(8 \times 2) =$	$16.$
"	" 10	"	" $=$	$(10 \times 0) =$	$0.$
"	" 12	"	" $=$	$-(12 \times 4) =$	$-48.$

Moment of the equilibrant, 4, at the unknown distance z from $M = 4 \times z = 4z$.

Then $4z - 24 + 16 + 0 - 48 = 0$, or $4z = 56$.

Hence, $z = 14$, that is, the point of application of the equilibrant is 14 units to the right from *M*; but 14 to the right from *M* is equal to 18 to the right from *K*, so the result reached is the same as before.

Compute the moments about *O*.

Will the direction, the magnitude, and the point of application of the resultant of the four forces be the same, no matter what point is taken to pass an axis through about which to compute moments?

Experiment 106. *To find what relations must exist among four forces (one acting north, one south, one east, one west) in one plane, in order that they may be in equilibrium.*

Apparatus. The same as in Exp. 103, but with one more spring balance.

Directions. Perform this experiment on a wide table. With the four spring balances, each pulling in a different direction, but always along some line marked on the board, as shown in Fig. 93, make three cases of equilibrium.

Take new points of application and new magnitudes of forces for each case. Record in a manner similar to that suggested in Exp. 103.

In each case, is there any fixed relation between the magnitude of a force acting in one direction and the magnitude of a force acting in the opposite direction?

Selecting one of the cases of equilibrium you have made, compute the moments of its forces with respect to one of the pegs used in this case. Then for this same case make three more computations, one with respect to each of the three remaining pegs. Finally, compute for this case the moments of its forces about some peg put at random into any hole in the board.

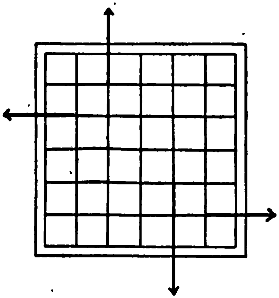


FIG. 93.

In each set of moments thus computed, does the algebraic sum of the moments equal zero?

Calling forces acting east positive, those acting west negative, forces acting north positive, those acting south negative, state carefully two general laws modeled after those given on page 261.

QUESTIONS. When any number of forces are acting upon a pivoted body, what is the necessary condition for their causing no rotation? When a body is subjected to forces acting in one plane only, what conditions must be fulfilled in order that the body may neither slide nor turn?

GRAVITY.

114. Center of Gravity. The earth pulls downward on each of the many particles of which a body is com-

posed. This downward pull of the earth is called *gravity*. In the next experiment we shall take as the body to be considered a piece of cardboard. Since the cardboard is composed of a great number of particles, there are a vast number of forces acting downward upon the body. All these forces are practically parallel. (Why?) The object of the experiment is to find whether the resultant of these parallel forces passes through a fixed point about which the cardboard will balance, if supported at this point.

Experiment 107. *To find the center of gravity of a piece of cardboard of triangular shape.*

Apparatus. A triangle of cardboard whose sides, for example, are 10^{cm}, 20^{cm}, and 25^{cm} long; a piece of thread; a pin; a bit of sheet lead.

Directions. Through the cardboard, very near one corner, stick a pin, and enlarge a little the hole thus made, so that when the pin is stuck horizontally into the edge of the table, the cardboard will swing freely with but little friction at the hole. When the cardboard, thus suspended, comes to rest, "the resultant force of its weight is balanced by the upward elastic resistance to bending exerted by the pin; and a plumb line, made of a bit of lead fastened to the end of a piece of thread, hung over the pin, will give the direction in which either force acts." Hang the plumb line over the pin by a loop, as shown in Fig. 94. Have the plumb line long enough to reach more than across the cardboard. Mark the point where the plumb line crosses the lower edge of

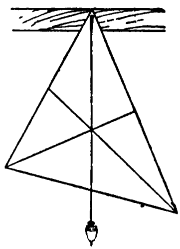


FIG. 94.

the cardboard. Now take down the cardboard and draw on it, with a sharp-pointed lead-pencil, a line from the pin hole to the point where the plumb line crosses the edge of the cardboard; this line, thus drawn, gives the direction of the resultant force of gravity on the cardboard.

Hang the cardboard from another corner, and repeat the process. A second line for the direction of the resultant force of gravity is thus obtained. Call the point in which the second line cuts the first one, G. Now suspend the cardboard from its third angle.

Does the plumb line cross the point G?

Lay the cardboard in a horizontal position on a pin-point, so that G shall rest upon the pin-point.

Does the card balance?

Does the resultant of all the parallel forces constituting the weight of a body pass through a fixed point?

Why is this point called the "center of gravity"?

Does the center of gravity of the piece of cardboard lie on one of the surfaces, or midway between the two surfaces?

Can we regard the weight of a body as collected at its center of gravity (that is, if we could replace all the parallel forces which constitute the weight of the body by a single force equal to the sum of this great number of parallel forces, would the line of action of this single force pass through the center of gravity of the body)?

This question we shall try to answer in the next experiment.

Experiment 108. *To find whether the weight of a body acts just as if it were all collected at the center of gravity of the body.*

Apparatus. A wooden stand like that used in Exp. 24; a heavy iron ball; a triangular prism of wood; a 30-pound spring balance.

Directions. Record the weight of the ball in pounds, and also that of the stand. On a table, near the corner, lay the triangular piece of wood, and on this lay the stand (Fig. 95) to which the iron ball is hung by a loop of string. Have the edge, not the face, of the meter stick, which is fastened to the stand, rest on the triangular piece of wood. On the triangular piece of wood balance the stand with the weight attached. Then record the distance from the end, A (Fig. 95), to the point of application, P, of the weight; also record the distance from A to the fulcrum, S.

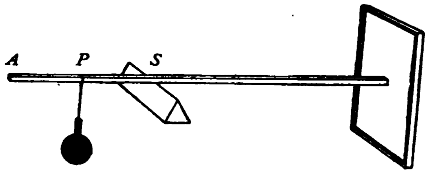


FIG. 95.

What is the distance in centimeters from P to S?

Considering the weight of the ball as one downward force, and the weight of the stand, the body, as the other downward force, find, by applying the principle of moments, at what distance from the fulcrum, S, the weight of the stand, if collected at one point, would have to be applied in order to produce the state of equilibrium that is observed. Apply the principle of moments in the following way: Multiply the weight of the ball by its distance from the fulcrum; also, denoting by x the distance from the fulcrum to the point where the weight of the stand would have to be collected in order to produce the state of equilibrium observed, multiply the weight of the stand by x. Put the two products thus obtained equal to each other

(we have a right to do this by the second condition of equilibrium, page 261), and solve the equation thus obtained to get the value of x. Add to x the distance, SA, from the fulcrum to the end, A, of the stand.

What is the distance from A of the point obtained by this computation?

Now change the position, P, of the ball on the stand, and balance again. Make measurements and record as before.

What is the distance from A of the point obtained by this computation?

Finally, remove the ball, and balance the stand alone on the triangular piece of wood.

What is the distance in centimeters from the end, A, to the balancing-point, the center of gravity, the present position of the fulcrum?

How does the distance just obtained compare with the two computed distances from A?

Has or has not the weight of the stand acted as if collected at the center of gravity?

How great a pressure has been exerted upon the support, that is, upon the triangular piece of wood, in each of the three cases?

Can you always regard the weight of a body as a force applied at the center of gravity of the body?

How could you find, by applying the teachings of this experiment, the weight of the stand, if the position of its center of gravity, the position of the fulcrum, the position of the iron ball, and also the weight of the iron ball were given?

How could you find by experiment the center of gravity of the stand?

EXAMPLES.

1. A uniform straight lever 10 ft. long balances at a point 3 ft. from one end, when 12 lbs. are hung from this end and an unknown weight from the other. The lever itself weighs 8 lbs. Find the unknown weight.

Solution. Denoting by x the unknown weight in pounds, and regarding the weight of the lever as a force applied at its center of gravity, take moments about the point of support thus:

$$12 \times 3 = 8 \times 2 + x \times 7,$$

or

$$7x = 36 - 16 = 20.$$

$$\therefore x = 2\tfrac{6}{7}.$$

Hence, the weight which was to be found is $2\frac{6}{7}$ lbs.

NOTE. Whenever we say a lever is uniform, we mean that its center of gravity is the center of its length.

2. A straight lever 6 ft. long weighs 10 lbs., and its center of gravity is 4 ft. from one end. What weight at this end will support 20 lbs. at the other, when the lever is supported at a distance of 1 ft. from the end nearer the center of gravity?

3. A telegraph pole is made of three hollow iron cylinders joined end to end. Each cylinder is 4^{m} long; the lowest weight 250^{kg}, the middle one 150^{kg}, and the uppermost 50^{kg}. Find the center of gravity of the pole.

SUGGESTIONS. Regard the weight of each section as applied at the center of gravity of the section to which it belongs. Imagine the pole placed in a horizontal position. Find where a prop would have to be placed to produce equilibrium.

4. A cube of wood, 10^{cm} on each edge and of specific gravity 0.5, is covered on one side by a plate of metal 10^{cm} square, 1^{cm} thick, of specific gravity 5. How far from the outer surface of the metal plate is the center of gravity of the whole?

5. Two uniform cylinders of the same diameter, whose lengths are 1 ft. and 7 ft., respectively, and whose weights are in the ratio of 5 to 3, are joined together so as to form one cylinder. Find the position of the fulcrum about which the whole will balance.

LEVERS.

115. Classes of Levers. A lever is a rod, or bar, by means of which a force can be applied more advantageously than it otherwise could in moving a body. A crowbar is a lever. It is rested, near one end, on a piece of stone or other firm support, called a *fulcrum.* The end near the fulcrum is placed under the body to be raised, and a force is applied at the other end tending to bring this end downward. The body to be moved is called the *weight*, and the force applied at the other end of the crowbar is called the *power.* There are three classes of levers. A lever like the one just described, where the *fulcrum* is *between* the weight and the power, is called a lever of the *first class.*

An oar, when used in rowing a boat, is a lever. The water into which the blade of the oar dips is the fulcrum, the resistance of the boat to being urged forward is the weight, which meets the oar at the rowlock, and the force of the rower's arm is the power. When the *weight* is *between* the fulcrum and the power, as in the case of an oar, the lever is called a lever of the *second class.*

A pitchfork, when used in lifting hay, is a lever. One end is held firmly in the hand, which is the fulcrum, the hay at the other end is the weight, and the force, applied by the other hand to the pitchfork somewhere between the fulcrum and the weight, is the power. When the *power* is *between* the fulcrum and the weight, the lever is called a lever of the *third class.*

In many problems concerning levers, the weight and the power are supposed to act at right angles to the lever,

and the distance measured along the lever from the fulcrum to the weight is called the *weight-arm*, while the distance from the fulcrum to the power is called the *power-arm*. By applying the principle of moments we have:

$$\text{power} \times \text{power-arm} = \text{weight} \times \text{weight-arm}.$$

QUESTIONS. Which is the greater, the power or the weight in a lever of the first class? In a lever of the second class? In a lever of the third class?

NON-PARALLEL FORCES.

116. Concurrent Forces. We have given considerable study to cases of equilibrium where we have had three *parallel* forces acting on a body, and we have found the resultant and the equilibrant of groups of parallel forces.

It now remains for us to study the conditions of equilibrium of three *concurrent* forces, that is, forces whose lines of action meet in a point.

Experiment 109. *To find the condition for equilibrium of three concurrent forces.*

Apparatus. Three 30-pound spring balances; two pieces of slender, strong, hard-twisted string 70^{cm} or 80^{cm} long; a wooden block like the one whose specific gravity you found; a meter stick.

Directions. The experiment is to be performed on a table with a smooth top. Note the error of each balance when in the horizontal position. Tie a loop in each end of one string. Do the same to the other string, only make one of the loops 10^{cm} long. Pass the first-mentioned string through the large loop, so that the large loop lies between the loops in the ends of this string, each of which loops should be slipped on the hook of a spring

balance, one loop only on one hook. The spring balances should be kept in place by having their rings fastened to the screws in blocks clamped near two corners of the table, shown in Fig. 96, but the balances should be able to swing freely round the screws in any horizontal position. Slip the remaining loop of the second string over

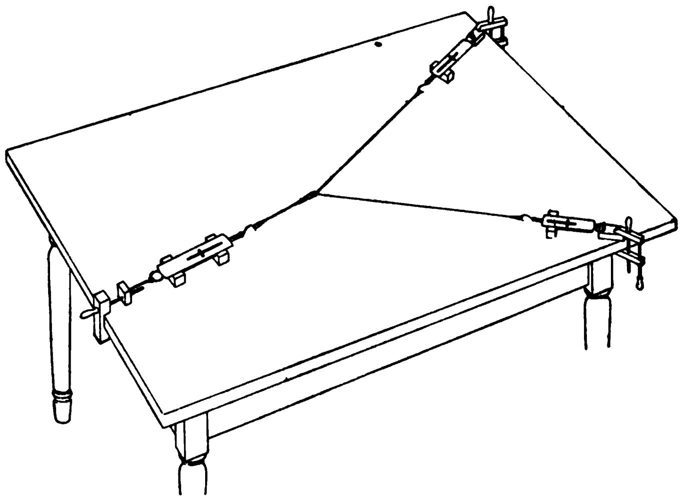


FIG. 96.

the hook of the remaining spring balance. Now pull the third spring balance by its ring in any desired direction and with such a force as to draw the strings tight, and make each balance point in the same direction as the string attached to its hook. By means of a clamp, fasten the balance by its ring. Make a careful reading of each balance, taking care to apply the proper correction to each. Put the open note-book beneath the junction of the strings, and on the page, with the block as a guide,

draw, with a sharp lead-pencil, a line a few centimeters in length parallel to each string and beneath it. From the point where these lines meet, when produced if necessary, lay off on each line an arrow proportioned in length to the reading of the balance whose pull was directed along the line, the tip of the arrow in each case pointing away from the junction. From the tip of *any* arrow lay off a line *parallel* and *equal* to one of the other arrows, and from the end of the line thus drawn draw another line parallel and equal in length to the remaining arrow. (It is not impossible that the line last drawn will coincide with the line to which it is drawn parallel.)

What is the name of the geometrical figure formed by the two lines just drawn and the selected arrow.

Point out how the following statement agrees with the result of the experiment:

If three forces acting upon a material point are in equilibrium, and three lines be drawn, without taking the pencil off, parallel to the successive forces acting on the point and in the same direction with them, and proportional to them in magnitude, a triangle will be formed.

The triangle thus formed is called the *triangle of forces.*

If the three forces are not in equilibrium, the lines will not form a triangle.

TRIANGLE OF FORCES.

117. Principle of the Triangle of Forces. If *three* forces are in equilibrium, and any triangle be drawn with its sides parallel to the lines of action of the forces, the lengths of these sides will represent the relative magnitudes of the corresponding forces.

Test the statement just made by drawing a triangle on the page of your note-book, the sides of the triangle being drawn parallel to the direction of the forces recorded in Exp. 109.

On two of the arrows, in your original figure, as sides, construct a parallelogram. Produce the remaining arrow backwards by its own length from the point whence the three original arrows spring.

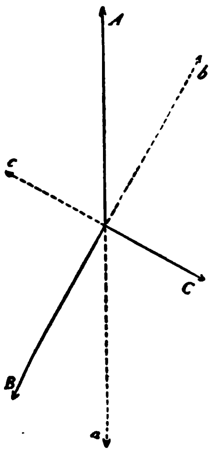


FIG. 97.

Does the line you have just drawn coincide or nearly coincide with the diagonal of the parallelogram?

In the case of three concurrent forces, as *A*, *B*, and *C* of Fig. 97, in equilibrium, we have the following relations:

A	is	the	equilibrant	of	*B*	and	*C*.
B	"	"	"	"	*A*	"	*C*.
C	"	"	"	"	*A*	"	*B*.
a	"	"	resultant	"	*B*	"	*C*.
b	"	"	"	"	*A*	"	*C*.
c	"	"	"	"	*A*	"	*B*.

Questions. What is the condition that must hold in order that three concurrent forces shall be in equilibrium?

A force of 10 lbs. acts due east, and a force of 10 lbs. acts due north, at the same point. What is the magnitude of their resultant? Of their equilibrant? In what direction does the resultant act?

EXAMPLES.

1. A ball, of weight 2 lbs. and of radius 4 in., is suspended by a string fastened to a vertical wall. The ball rests against the wall, and the direction of the string passes through the center of the ball. If the distance of the point at which the string is fastened to the wall is 6 in. above the

point where the ball touches the wall, find the pressure of the ball against the wall.

Solution. In Fig. 98 let P be the point at which the end of the string is fastened to the wall; let M be the point at which the ball touches the wall. We are asked to find the pressure of the ball against the wall. This pressure will be equal to the push of the wall against the ball. There are three forces which keep the ball in equilibrium, the weight of the ball, the push of the wall, and the tension of the string; but these three forces pass through a single point, C, so let us denote the weight by CW, the push by CR, and the tension by CT. These three forces are in equilibrium, and are parallel, respectively, to PM, MC, and CP, the sides of the right triangle, PMC. Hence, by the principle of the triangle of forces,

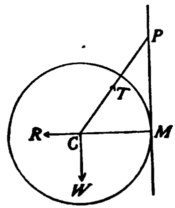


FIG. 98.

$$\frac{CW}{CR} = \frac{PM}{MC}.$$

By the conditions of the problem, $CW = 2$ lbs., $PM = 6$ in., and $MC = 4$ in., consequently, if we denote by x the required force, CR,

$$\frac{2}{x} = \frac{6}{4}$$

$$6x = 8$$

$$\therefore x = 1.33.$$

Hence, the pressure of the ball against the wall is 1.33 lbs.

2. Find the tension of the string in Ex. 1.

3. A picture frame weighing 10 lbs. is hung by a cord passing over a nail, the two parts of the cord making an angle of 90° with each other. Find the tension in the cord.

4. A uniform pendulum rod is pulled aside from the vertical by a horizontal force applied to the lower end. If the pendulum rod weighs 10^g, what must be the horizontal force in order that the pendulum rod, when in equilibrium, shall make an angle of 45° with its former position?

5. An isosceles triangle, the vertical angle of which is equal to 120°, is drawn on a vertical wall with its base horizontal and its vertex downwards. At each corner of the triangle a smooth peg is driven into the wall at right angles. A thread with 12^g attached to each end is passed under the lower peg and over the other two pegs. Find the pressure on each peg. Find also the vertical and the horizontal pressure on each peg.

COMPOSITION AND RESOLUTION.

118. Composition and Resolution of Forces. The process of finding the *resultant* of a group of forces is called the *composition of forces*, while the process of finding *two* forces which will exactly replace a *single* force in its action upon a body is called the *resolution of forces*.

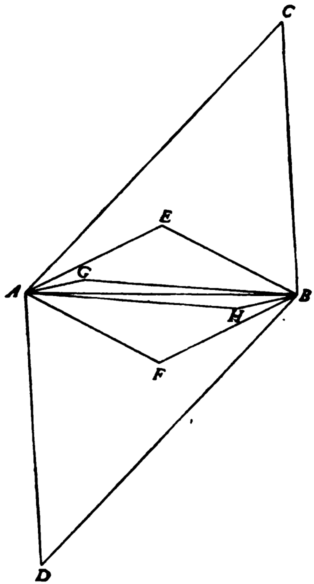


FIG. 99.

In the composition of forces we get one definite answer to the problem.

In the resolution of forces we get an indefinite number of answers, for upon a given force, *AB*, as diagonal we can construct an indefinite number of parallelograms, a few of which are shown in Fig. 99. The two forces into which a single force is resolved are called *components*. It is often convenient to resolve a force into two components at right angles to each other.

FRICTION.

119. Friction a Force; Coefficient of Friction. Friction is the force which opposes the sliding of one body over another.

If R denotes the *perpendicular* pressure between two surfaces which slide over each other, and F denotes the friction, the quotient of $\frac{F}{R}$ has been found to be a constant number for any two given surfaces, provided the corresponding surfaces are of the same material and in the same condition, and is called the *coefficient of friction.*

The coefficient of friction is denoted by the Greek letter μ, that is, $\frac{F}{R} = \mu$, and consequently $F = \mu R$.

Does the last equation suggest to you the propriety of calling μ the "coefficient of friction"?

Experiment 110. *To find the coefficient of friction of wood sliding on paper.*

Apparatus. A smooth board, about 50^{cm} long; a block of wood such as you got the specific gravity of; an 8-ounce spring balance; a sheet of paper about 25^{cm} long and a little wider than the board.

PART 1. Where the sliding is on a horizontal plane.

Directions. Find what fraction of an ounce it takes to draw the pointer down to the zero line, when the balance is in a horizontal position. Spread the paper smoothly on the board, fold the edges of the paper over the sides of the board, and fasten by tacks. Lay the board horizontally. Place the block on one of its narrowest sides on the paper. Tie a thread round the block, but in such a way that no part of the thread shall come between the block and the paper. Fasten the end of the thread to the hook of the spring balance, and pull with the spring balance held in a horizontal position with a

force sufficient to keep the block moving with uniform velocity along the paper. Record the force indicated by the balance, and correct this force for the error of the balance when held in the horizontal position.

Then, taking care as before that the thread does not lie between the block and the paper, lay the block on its broad side, and record as before the force necessary to maintain uniform motion.

From the two cases just tried, should you say that the amount of friction depends upon the extent of surface in contact?

Now load the block, still on its broad side, with 200^g and 400^g in turn, recording in each case the force required to maintain uniform motion. Weigh the block itself in ounces; and also find the weight in ounces of the two weights, the 200^g and the 400^g. Then calculate the coefficient of friction for each of the four cases tried. To compute the coefficient of friction, divide the friction (the number of ounces the spring balance indicated when corrected for the zero error) by the perpendicular pressure (the weight of the block in ounces plus the weight in ounces of any load put upon the block).

What value do you get for the coefficient of friction?

PART 2. Where the sliding is on an inclined plane.

Raise one end of the board until the unloaded block, once started, will slide along the paper with uniform velocity. Then, keeping the board fixed in this position, measure the vertical distance from the *under* (Why?) side of the raised end to the table, and the horizontal distance from the foot of this vertical line to the point where the

lower end of the board rests upon the table. The coefficient of friction can be calculated by dividing the vertical distance by the horizontal.

Does the numerical value which you get equal that which you got by the other method?

NOTE. The angle which the inclined plane makes with the horizontal plane, when the body is just about to slide, is called the *angle of repose*, or the *angle of friction*.

The proof of the statement that the coefficient of friction can be obtained by dividing the vertical distance by the horizontal is given in the demonstration of the following:

Theorem. *When a body slides with uniform velocity down an inclined plane, the coefficient of friction is equal to the height of the plane divided by the base.*

Let O (Fig. 100) be a body sliding with uniform velocity down the inclined plane, BA.

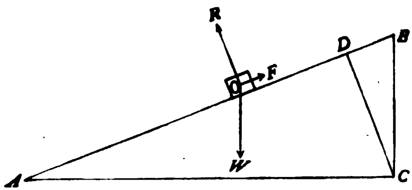


FIG. 100.

To prove $\mu = \frac{CB}{AC}$:

Let OW, OF, and OR represent weight of O, friction, and perpendicular pressure respectively.

From C draw $CD \parallel OR$.

CD, DB, BC are $\parallel$ resp. to OR, OF, OW.

$\therefore \frac{DB}{CD} = \frac{OF}{OR}$ (by the principle of the $\triangle$ of forces).

$\triangle ABC$ is a rt. $\triangle$.

$CD \perp AB$ (by the principle that a st. line $\perp$ to one of two $\|$'s is $\perp$ to the other).

$\therefore$ $\triangle$ CBD and ABC are similar.

$$\therefore \frac{CB}{AC} = \frac{DB}{CD}.$$

But $$\frac{OF}{OR} = \frac{DB}{CD},$$

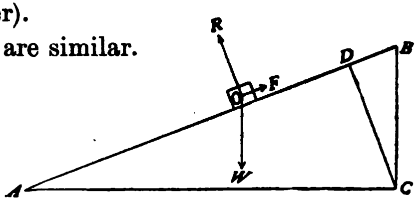


FIG. 100.

And $$\mu = \frac{OF}{OR} \text{ (by definition).}$$

$$\therefore \mu = \frac{CB}{AC}.$$

EXAMPLES.

1. The coefficient of friction for wood on wood is 0.33. How great a force must be applied to a block of wood weighing 100 lbs. in order to drag it along a wooden plank ?

2. A body slides with uniform velocity down a plane inclined at an angle of 30° to the horizon. What is the coefficient of friction between the body and the plane ?

HINT. The hypotenuse is twice the length of the side opposite the angle of 30°.

3. A body, the weight of which is 10 lbs., rests upon a plane inclined at an angle of 30° to the horizon ; find the pressure at right angles to the plane, and the force of friction exerted.

4. If the height of an inclined plane be to the length as 3 is to 5, and a body the weight of which is 15 lbs. be supported by friction alone, find the force of friction in pounds.

WORK.

120. Work defined ; Unit of Work. One important object of the next experiment is to illustrate and enforce the meaning of the term *work* as used in physics. *Work is the overcoming of resistance* through space.

According to the meaning given in physics to the term *work*, the mere supporting of a book by the hand is *not work;* but if the book is raised, *work* is done, for a resistance, the weight of the book, is overcome (through a space).

The *unit of work*, among English-speaking engineers, is the *foot-pound* (ft. lb.). *The foot-pound is the amount of work done in overcoming the resistance of a force of one pound through a distance of one foot.*

If you should lift a pound-mass vertically a distance of one foot, the amount of work done would be one foot-pound.

QUESTIONS. How many foot-pounds of work must be done in dragging a 20-lb. weight a distance of 15 ft. along a horizontal plane, if the coefficient of friction is 0.4? If the coefficient of friction is 0.5?

Experiment 111. *To find the amount of work done in dragging a body up an inclined plane.*

Apparatus. An inclined plane, a stout plank with a long slot cut through it, one end resting on the floor, the other end raised above the floor, as shown in Fig. 101; a 4-pound spring balance; a small, strong carriage; four 200^g weights, with which to load the carriage.

PART 1. To find the work done, apart from that employed in overcoming friction, by a force parallel to the incline, in drawing the loaded carriage (Fig. 101) such a distance up the plane as to make the vertical rise 2 ft.

Directions. Along the incline measure the distance, *AB*, corresponding to a vertical rise of 2 ft.; then find the force parallel to this incline which would be necessary to move the loaded car with uniform velocity up the incline, if there were no friction. To determine and eliminate the friction at any part of the incline, find the

pull on the balance parallel to the incline required to move the carriage at a uniform velocity up the incline at this part, and then the pull in the same direction which will allow the carriage to move with uniform velocity down

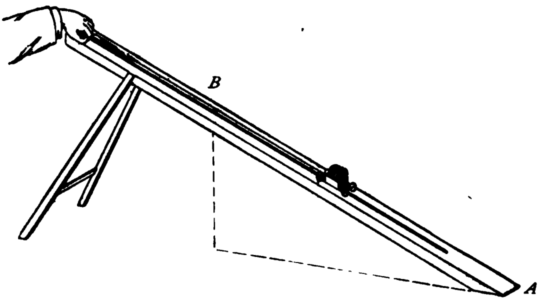


FIG. 101.

the incline at the given part. The average of the two pulls will be the pull that would be required if there were no friction, and half the difference between the two forces will be the force required to overcome the friction at the spot in question. The diagrams and the following discussion will, perhaps, make this clearer.

In the first diagram (Fig. 102) the car is supposed to be moving up the incline. The friction, F, and the resolved part, x, of the force of gravity oppose the motion. If the spring balance reads a lbs., we have

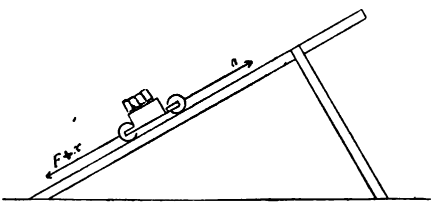


FIG. 102.

$$F + x = a. \qquad (1)$$

In the second diagram (Fig. 103) the car is supposed to be moving down the plane. The friction, F, acts in a direction opposite to the motion, so we have, if the balance now registers b lbs., $x = F + b$, or,

$$-F + x = b. \quad (2)$$

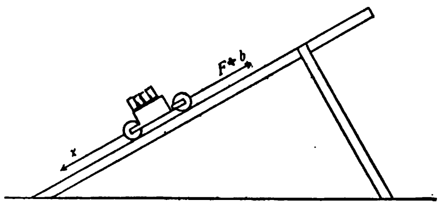


FIG. 103.

By adding (1) and (2) we have,

$$2x = a + b,$$

$$\therefore x = \frac{a+b}{2}.$$

Hence, the force necessary to keep the car moving up the incline, if we leave friction out of account, is $\frac{a+b}{2}$ lbs.

By subtracting (2) from (1) we find, $F = \frac{a-b}{2}$. If the plane were uniform, it would be necessary to determine the force at one place only; but as the incline may vary slightly, it is best to determine the force at three or four places and take the average.

Multiply this average force, x, by the distance, measured along the plane through which the force would have to act in order to draw the car up the plane so that its vertical rise would be 2 ft.

How many foot-pounds of work are required?

PART 2. To find the work, friction aside, done by a force acting parallel to the base of the incline, in drawing the loaded car such a distance up the incline as to make the vertical rise 2 ft.

Directions. Let the string that attaches the carriage to the balance pass through the slot cut in the plane, as shown in Fig. 104. Then draw the carriage up the plane, always pulling horizontally on the balance. Do not let the string rub on the sides of the slot. By observing the

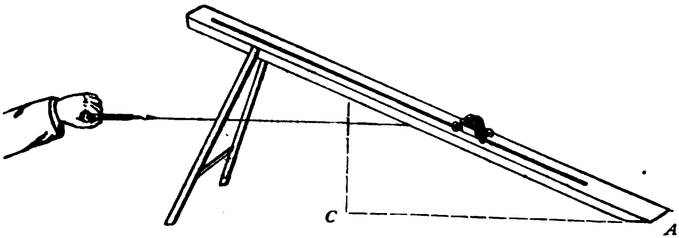


FIG. 104.

pull going up and the pull coming down, eliminate friction, as in Part 1. Measure the horizontal distance, AC, corresponding to a vertical rise of 2 ft. Multiply the force by the distance, AC.

How many foot-pounds of work are necessary?

PART 3. To find the amount of work done in raising the carriage and contents through a vertical distance of 2 ft.

Directions. Weigh the carriage and its load together on the same balance.

How many foot-pounds of work would it take to raise the carriage and its load through a vertical distance of 2 ft.?

How do the amounts of work in the three cases compare with each other?

121. Law of the Inclined Plane. The teachings of the preceding experiment lead to a very important result, known as the *law of the inclined plane.*

The amount of work done in dragging a body up an inclined plane is equal to the weight, W, of the body multiplied by the height, CB, of the plane (Fig. 105), whether the force used to drag the body acts parallel to the incline, AB, or parallel to the base, AC, consequently, if P denotes the force parallel to AB necessary to drag the body, and Q the force parallel to AC,

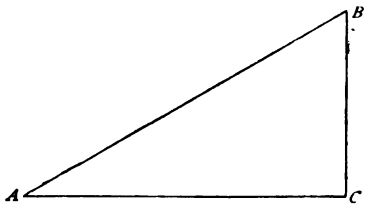


FIG. 105.

$$P \times AB = W \times CB,$$
$$Q \times AC = W \times CB,$$

that is, *the force required to drag the body multiplied by the distance through which it acts is equal to the weight of the body multiplied by the vertical distance through which it is raised.*

EXAMPLES.

1. A body weighing 100 lbs. rests upon an incline. (*a*) How much work must be done in drawing the body 10 ft. up this incline, the vertical ascent being 6 ft.?

Solution. $100 \times 6 = 600$ ft. lbs.

(*b*) What force is necessary to draw the body up the incline, if applied parallel to the incline?

Solution. $P \times 10 = 100 \times 6$; hence, $P = 60$ lbs.

(*c*) What force is necessary to draw the body up the incline if applied parallel to the base?

Solution. $Q \times \sqrt{10^2 - 6^2} = 100 \times 6$; hence $8Q = 600$, $\therefore Q = 75$ lbs.

(*d*) How much work must be done if the coefficient of friction is 0.2?

Solution. $100 \times 6 + 80 \times 0.2 \times 10 = 760$ ft. lbs.

NOTE. In the solution of (*d*), the expression $80 \times 0.2 \times 10$ is obtained by applying the principle of the triangle of forces.

2. A block weighing 10 lbs. rests on an incline such that the block must move 5 ft. in order to rise 3 ft. The pressure of the block against the incline is in this case 8 lbs. The coefficient of friction is 0.25. How much work must be done against gravity by a force parallel to the incline in drawing the block up 5 ft. How much work against friction?

3. An inclined plane rises 3.5 ft. for every 5 ft. of length. Find what force in pounds must be applied parallel to the incline to drag up a weight of 200 lbs.

4. A railway train weighing 30 tons is drawn up an inclined plane of 1 ft. in 60 by means of a rope and a stationary engine; find what number of pounds at least the rope should be able to support.

5. If the base of an inclined plane is to the height as 24 is to 7, find the force necessary to drag a body weighing 48 lbs. up the plane.

(*a*) When the force acts parallel to the incline.

(*b*) When the force acts parallel to the base.

6. While a car moves along a track a distance of 10 ft., a man pushes with a force of 25 lbs. against the side of the car in a direction at right angles to the track. Does the push of the man on the car assist its motion? Does his push, except by possibly increasing the friction, retard the motion of the car? How many foot-pounds of work does the man do upon the car during its motion?

INERTIA.

122. Inertia a Property of Matter. There is an inherent property of matter called *inertia*, which makes the application of a force necessary to bring about any change in the direction or magnitude of a body's motion.

To start a body requires the application of force; to stop a body requires the application of force; to deflect a moving body from its path also requires the application of force.

In brief, to start a body, to stop it, to increase its velocity, to diminish its velocity, or to deflect it from its path,

the application of a force is required. The property of matter that makes this application of force necessary is called *inertia.*

To get the quantity of matter in a body, we have compared the body, by means of a pan balance, with a standard mass. Since the attraction of the earth becomes greater as we approach the polar regions from the equatorial, a mass of matter would weigh more at the pole than at the equator. The spring balance does not measure the mass, that is, the quantity of matter a body contains, but it does measure the force of gravity at different parts of the earth's surface.

123. Definition of the Equality of Two Masses. In our discussion of the term *mass* in the early part of this chapter we tacitly assumed that two bodies have the same mass, no matter how much they may differ in substance, provided they are in equilibrium when placed in the opposite pans of a balance.

The idea of inertia, however, gives us a means of comparing two masses for equality without the use of a balance of any description. The method holds not only for the earth, but for all the heavenly bodies also, as well as for all regions of space. In the following experiment we shall apply this "inertia test" for the equality of two masses.

Experiment 112. *To find, without the use of a balance, whether two masses are equal or not.*

Apparatus. The mass-tester (Fig. 106), consisting of a vertical axis with two horizontal arms to carry cups (nickel-plated calorimeters), and a spiral spring with one end fastened to the axis and

the other fastened to the support in which the upper end of the axis is pivoted; two 100g weights; some lead shot; a platform balance.

Directions. In each of the cups put a 100g mass, turn the axis through 180°, and find accurately the time of six vibrations. Then take out the 100g mass and put in place of it lead shot; try to have equal amounts of shot in each of the cups so that the axis shall not bind where it is pivoted. By varying the amount of shot (adding more if the apparatus moves too fast, taking out some if the apparatus moves too slowly), get the apparatus to make the same number of vibrations in the same time as when the iron occupied the place of the lead. Find how many grams the shot weighs.

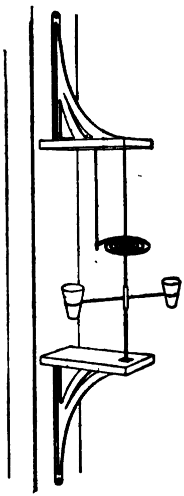


FIG. 106.

VELOCITY.

124. Measure of the Velocity of a Body. Whenever a body moves (that is, changes its position) it either passes over equal spaces in equal times, like the hand of a clock, and is said to have *constant* velocity, or else passes over unequal spaces in equal times, like a stone falling from a height toward the ground. From the instant the stone begins its downward motion, it falls with greater and greater swiftness until it rests upon the ground. When the body passes over unequal spaces in equal times, its velocity is said to be *variable.*

Definition. *Velocity is the rate of motion of a body.*

The velocity of a body is measured by the distance traversed divided by the time that elapses during the journey.

If we denote by s the distance traversed in time, t, by a body moving with constant velocity, v, we have

$$v = \frac{s}{t}. \qquad (1)$$

EXAMPLES.

1. If the tip of a clock-hand moves a distance of 30^{cm} in 20 minutes, what is the velocity?

Solution. We are to find the number of units of length traversed in a unit of time. If we take as our unit of length the centimeter, and as our unit of time the minute, the required velocity will be $\frac{30}{20} = 1.5^{cm}$ per minute.

If we take as our unit of length the centimeter, as before, and for our unit of time the second, the required velocity will be $\frac{30}{20 \times 60} = \frac{1}{40} = 0.025^{cm}$ per second.

2. Is a velocity of $\frac{3}{2}^{cm}$ per minute the same as a velocity of $\frac{1}{40}^{cm}$ per second?

3. If a tortoise walk at the rate of 3 in. a second, how many feet will he go in an hour?

4. How many seconds would a train take to go 0.2 of a mile at the rate of 20 miles per hour?

5. With what velocity must a horse trot in order to go 780 yds. in 1 minute?

ACCELERATION.

125. Measure of Acceleration. When a body is moving with a variable velocity, the rate at which the velocity changes is called *acceleration*. It is measured by the

velocity gained by the body in a certain time, divided by the time taken to gain it.

Acceleration is said to be *constant* when the velocity gains equal additions of acceleration in equal times.

Variable acceleration, which we shall have no occasion to consider, results when the velocity gains unequal additions of acceleration in equal times.

For each second the acceleration of a body falling in a vacuum near the surface of the earth is constant, and is equal to about 980^{cm} per second.

If a body moving with a velocity which has a constant acceleration, a, has a velocity of v_0 at the beginning of the time, t, and a velocity of v_1 at the end of this time, we have

$$a = \frac{v_1 - v_0}{t}. \qquad (2)$$

EXAMPLES.

1. If the velocity of a falling body after 3 seconds is 2840^{cm} per second, what is its acceleration?

Solution. We are to find the number of units of velocity acquired in a unit of time. If we take as our unit of velocity the centimeter per second, the acceleration in 1 second will be

$$\frac{2840}{3} = 980^{cm} \text{ per second.}$$

2. If a body is thrown downward with an initial velocity of 100^{cm} per second from a balloon, and after the lapse of 3 seconds its velocity is 2940^{cm} per second, what is its acceleration?

Solution. As acceleration is the gain in velocity divided by the time taken, in this case 3 seconds, in making this gain we have for the acceleration in 1 second

$$\frac{2940 - 100}{3} = \frac{2840}{3} = 980^{cm} \text{ per second.}$$

3. A body starts from rest and acquires a velocity of 1200^{cm} per second in half a minute; what is its acceleration?

4. A body starts with a velocity of 75^{cm} per second, and 5 seconds afterward is moving with a velocity of 97^{cm} per second; what is its acceleration in 1 second?

5. Starting with a velocity of 25^{cm} per second, a body for every second it moves has an acceleration of 980^{cm} per second; what will be its velocity in 1, 2, 3, and 6 seconds, respectively?

DISTANCE TRAVERSED.

126. Distance traversed by a Moving Body. If the initial velocity of a moving body is v_0, and the velocity it has at the end of the time, t, is v_1, its average increase of velocity will be

$$\frac{v_1 - v_0}{2},$$

and denoting by s the distance passed over by the body by reason of the increase of velocity from v_0 to v_1, we have

$$s = \frac{v_1 - v_0}{2} \times t. \qquad (3)$$

But $$v_1 - v_0 = at \text{ by (2)},$$

$$\therefore s = \frac{at^2}{2}.$$

EXAMPLES.

1. How far will a body fall in 2 seconds?

Solution. The relation existing between the time during which a body moves, under a constant acceleration, and the distance traversed by the body in this time is

$$s = \frac{at^2}{2},$$

$$\therefore s = \frac{980 \times 2^2}{2} = 1960^{cm}.$$

2. How much farther will a body fall during the second second of its fall than during the first?

Solution. The distance fallen in 2 seconds $= \dfrac{980 \times 2^2}{2} = 1960^{cm}$.

" " " " 1 " $= \dfrac{980 \times 1^2}{2} = 490^{cm}$.

Hence, the distance traversed during the second second will be

$$1960^{cm} - 490^{cm} = 1470^{cm}.$$

3. Show that the distance passed over in the $(t + 1)$ second by a body acted upon by a constant acceleration, a, is $\frac{a}{2}(2t + 1)$.

Solution. Distance traversed in t seconds $= \dfrac{at^2}{2}$.

" " " $(t + 1)$ " $= \dfrac{a(t+1)^2}{2}$.

Hence, the distance traversed in the $(t + 1)$ second is

$$\frac{a(t+1)^2}{2} - \frac{at^2}{2} = \frac{a}{2}[(t+1)^2 - t^2] = \frac{a}{2}(t^2 + 2t + 1 - t^2) = \frac{a}{2}(2t + 1).$$

4. Show that the distance passed over during the 2 seconds immediately following the tth second by a body starting from rest and moving under a constant acceleration, a, is $2a(t + 1)$.

5. How many seconds will a falling body, starting from rest, require to fall 490^{cm}?

6. Two bodies are let fall from the same point at an interval of 2 seconds; find how many centimeters they will be apart at the end of 6 seconds from the fall of the first, the acceleration of gravity being 981^{cm} per second.

QUANTITY OF MOTION.

127. Momentum. If two bodies, one having twice the mass of the other, are moving with equal velocity, the greater mass has the greater quantity of motion. If the bodies of equal mass are moving with unequal velocities, the body moving with the greater velocity has the greater quantity of motion. The quantity of motion, then, which a body has, depends upon the mass of the body and upon the velocity with which it is moving; so we define the

momentum (which means, simply, quantity of motion) of a body as the product of the mass of the body and its velocity. If the mass of a body is m and its velocity v, its momentum is mv.

The meaning of momentum will be illustrated by the next experiment, which has for its object the comparison of the total amount of momentum of two masses before colliding with each other, and the total amount of momentum of the two masses after collision.

Experiment 113. *To find how the total amount of momentum of two masses before collision compares with the total amount of momentum after collision.*

Apparatus. Two ivory balls, one weighing about 50^g, the other about 200^g; a flat bar of wood about 50^{cm} long, and of a width somewhat greater than the sum of the radii of the balls; a piece of apparatus like that shown in Fig. 107; fine iron wire, No. 33 B. & S.

PART 1. When the larger ball strikes the smaller at rest.

Directions. Fasten the flat bar, in a horizontal position, as high above the floor as practicable. Into each end of the bar drive two tacks, whose distance apart shall equal the sum of the radii of the balls. To one of these tacks fasten one end of a long piece of fine iron wire, pass the other end through a little screw-eye in one of the ivory balls. Then carry this end to the tack driven into the other end of the board directly opposite the first, and there fasten the wire, after drawing it up till the ball, when hanging freely, almost touches the center of the scale, which is shown in Fig. 107. The other ball should

be suspended in the same way. The positions of the balls should then be adjusted till their centers are in a horizontal line, and directly over the scale. Record the weight of each ball, and also the weight of the wire which supports each.

Allowing the smaller ball to hang at rest, draw the larger ball through an arc of about 10°, keeping it over

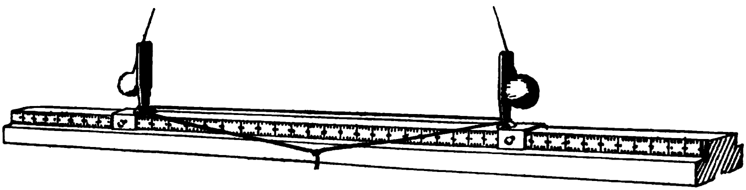


FIG. 107.

the scale. Record the reading on the scale directly beneath each ball. Then release the larger ball, and again observe and record the readings, when the balls have swung as far as they will after collision.

Through how many centimeters, as measured on the scale, did the larger ball move before collision?

Through how many centimeters did the larger ball move after collision?

Through how many centimeters did the smaller ball move after collision?

Make this trial several times, always drawing the larger ball through the same arc, and take the average of the corresponding readings in the several trials.

When experimenting with the simple pendulum, you found that the length of the arc through which the pendulum swung made no perceptible difference in the time

of an oscillation. In the present case, the velocity with which the larger ball is moving, when it *hits* the smaller, is proportional to the distance through which it has moved; also the velocity with which the smaller ball *starts* after it has been hit by the larger is proportional to the distance it moves until it starts to swing back; and the distance the larger ball moves after the collision is proportional to the velocity which it had just after the collision. As we wish only to compare the momenta of the balls before impact with the momenta after impact, we shall use these distances in place of the actual velocities to which they are proportional.

The following questions will show the line of reasoning by which the result is reached:

(1) What is the product of the mass of the larger ball plus half the mass of its supporting wire, and the distance through which it moved before striking the smaller?

(2) Calling the product, asked for in the preceding question, *momentum*, what was the momentum of the smaller ball before it was struck by the larger?

(3) What was the momentum of the larger ball after collision?

(4) What was the momentum of the smaller ball after collision?

(5) How does the sum of the answer to (1) and the answer to (2) compare with the sum of the answer to (3) and the answer to (4)?

Why was only half of the weight of the suspending wire added in each case to the weight of the ball?

PART 2. When the smaller ball strikes the larger at rest.

Directions. The apparatus should be arranged as for Part 1.

Proceed as in Part 1, but allow the larger ball to hang at rest, and draw the smaller one through an arc of about 10° and then release it. As before, repeat the trial and record the readings.

Through how many centimeters did the smaller ball move before collision?

Through how many centimeters did the smaller ball move after collision?

Through how many centimeters did the larger ball move after collision?

(1) What was the momentum of the smaller mass before collision?

(2) What was the momentum of the larger mass before collision?

(3) What was the momentum of the smaller mass after collision?

(4) What was the momentum of the larger mass after collision?

(5) How does the sum of the answer to (1) and the answer to (2) compare with the sum of the answer to (3) and the answer to (4)?

NOTE. When a ball, as the smaller one in this case, moves after collision in the opposite direction to that in which it was moving before collision, its velocity is negative, and therefore its momentum is also negative; so when we get the sum of the answers to (3) and (4), it is the *algebraic sum* which is required.

PART 3. When the balls, moving from opposite directions, meet at their point of rest.

Directions. Draw each of the balls through an arc of about 10° and release them, taking and recording as before the readings of their various positions.

Through how many centimeters did the larger ball move before collision?

Through how many centimeters did the smaller ball move before collision?

Through how many centimeters did the smaller ball move after collision?

Through how many centimeters did the larger ball move after collision?

Remembering that, if velocity in one direction is considered positive, velocity in the opposite direction is negative, find the algebraic sum of the momenta of the two masses before collision; also the algebraic sum of the momenta of the two masses after collision.

How does the first sum compare with the second?

EXAMPLES.

1. A ball weighing 20^g moving north with a velocity of 50^{cm} per sec. strikes centrally a ball weighing 100^g which is at rest. After the collision the larger ball moves north with a velocity of 12^{cm} per sec. What velocity has the smaller ball after the collision, and in what direction is it moving?

Solution. Let x denote the velocity of the smaller ball after the collision. We have

$100 \times 0 = 0$	momentum of	larger ball	before	collision.
$20 \times 50 = 1000$	" "	smaller "	"	"
$100 \times 12 = 1200$	" "	larger "	after	"
$20 \times x = 20x$	" "	smaller "	"	"

But the total momentum before collision is equal to the total momentum after collision; hence

$$0 + 1000 = 1200 + 20x$$
$$-20x = 200$$
$$\therefore x = -10.$$

That is, the smaller ball is moving after the collision with a velocity of 10^{cm} per sec. towards the south. The minus sign denotes that the direction of motion is opposite that which the ball had before the collision.

2. If a ball of mass 25^{g} moving with a velocity of 60^{cm} per sec. strikes centrally another ball at rest the mass of which is 100^{g}, and then *rebounds* with a velocity of 20^{cm} per sec., with what velocity does the larger ball move after the collision?

3. A ball of mass 200^{g} moving with a velocity of 30^{cm} per sec. strikes centrally another ball of mass 65^{g} at rest. After the collision the larger ball moves on with a velocity of 17^{cm} per sec. With what velocity does the smaller ball move after the collision?

4. A ball of mass 75^{g} moving north with a velocity of 200^{cm} per sec. strikes centrally another ball of mass 60^{g} moving south with a velocity of 100^{cm} per sec. After the collision the larger ball moves on with a velocity of 30^{cm} per sec. What is the velocity of the smaller ball after the collision, and in what direction is it moving?

ABSOLUTE UNIT OF FORCE.

128. The Dyne. The units of force which we have used in our work in the laboratory have been gravitational units, or units of force depending for their value upon the pull of gravity. As the force of gravity is not constant, but increases in strength as we go from the equator towards either pole, a unit of force has been adopted called the *dyne*, which is independent of the earth's attraction, and remains constant for all parts of the universe. The dyne is a unit of force which depends upon the *inertia* of matter. (See definition of inertia, page 286.)

Definition. *The dyne is that force which, acting upon a mass of 1^{g} for one second, gives it a velocity of 1^{cm} per second.*

If the dyne acts for 2 sec. on a mass of 1^{g}, the velocity imparted will be 2^{cm} per sec.; for 3 sec., 3^{cm} per sec.; and so on.

If a force of 2 dynes acts for 1 sec. on a mass of 1^{g}, the velocity imparted will be 2^{cm} per sec.; for 3 dynes, 3^{cm} per sec.

If the dyne acts on a mass of 2^{g} for 1 sec., the velocity imparted will be $\frac{1}{2}^{cm}$ per sec.; on a mass of 3^{g}, $\frac{1}{3}^{cm}$ per sec.

If f denotes the number of dynes, m the number of grams acted upon, t the number of seconds the action lasts, and v the velocity acquired,

$$v = \frac{ft}{m}. \qquad (4)$$

Equation (4) on clearing it of fractions takes the form

$$mv = ft.$$

This equation expressed in words is: "The momentum of a body is equal to the force acting upon it multiplied by the time the force has been acting."

ABSOLUTE UNIT OF WORK.

129. The Erg. Work has already been defined as the overcoming of resistance; and work is measured by the product of the force (used in overcoming the resistance) and the distance through which the force acts.

If the dyne be taken as the unit of force and the centimeter as the unit of distance, then the unit of work is the work done by one dyne acting through a distance of 1^{cm}. This unit of work is called the *erg* (from the Greek *ergon*, work). The *erg* is an absolute unit of work, while the *foot-pound* is a gravitational unit of work. The erg is a unit, the magnitude of which remains unchanged in whatever part of the universe it may be employed; while the foot-pound could be used nowhere except upon the surface

of the earth, and even then it would have different values at different parts of the earth's surface.

SYSTEMS OF UNITS.

130. The English System; the Centimeter-gram-second System. In the English System the foot is taken as the unit of length, the pound as the unit of mass, and the second as the unit of time, while in the Centimeter-gram-second System (often written in the abbreviated form C.G.S. System), the centimeter is taken as the unit of length, the gram as the unit of mass, and the second as the unit of time.

The following list of units with their definitions is important:

UNITS OF FORCE.

	ENGLISH SYSTEM.	C.G.S. SYSTEM.
Gravitation,	Pound,	Gram.
Absolute,	Poundal,	Dyne.

UNITS OF WORK.

	ENGLISH SYSTEM.	C.G.S. SYSTEM.
Gravitation,	Foot-pound,	Gram-centimeter.
Absolute,	Foot-poundal,	Erg.

The gravitation unit of force, the *pound*, is the pull of the earth upon a mass of one pound.

The absolute unit of force, the *poundal*, is that force which, acting upon the pound-mass for one second, will give it a velocity of one foot per second.

The gravitation unit of work, the *foot-pound*, is the amount of work done in lifting the pound-mass one foot high in opposition to the force of gravity; or, what is the

same thing, the foot-pound is the amount of work done in overcoming the force of one pound through a distance of one foot.

The absolute unit of work, the *foot-poundal*, is the amount of work done in overcoming a resistance of one poundal through a distance of one foot.

The gravitation unit of force, the *gram*, is the pull of the earth upon a mass of one gram.

The absolute unit of force, the *dyne*, is that force which, acting upon the gram-mass for one second, will give it a velocity of one centimeter per second.

The gravitation unit of work, the *gram-centimeter*, is the amount of work done in lifting the gram-mass one centimeter high in opposition to the force of gravity; or, what is the same thing, the gram-centimeter is the amount of work done in overcoming the force of one gram through a distance of one centimeter.

The absolute unit of work, the *erg*, is the amount of work done in overcoming a resistance of one dyne through a distance of one centimeter.

131. Energy. Energy is the ability of doing work, and is measured by the work done.

Thus, if a force of f dynes is overcome through a distance of s^{cm}, the work done $= fs$ ergs. fs is also the measure of the energy expended.

From (4) $$f = \frac{mv}{t}.$$

But $$s = \frac{1}{2} at^2.$$

$$\therefore fs = \frac{mv}{t} \times \frac{1}{2} at^2 = \frac{mvat}{2}.$$

But $$at = v.$$

$$\therefore fs = \frac{mv^2}{2}. \tag{5}$$

The quantity $\frac{mv^2}{2}$ is denoted by the expression "the kinetic energy of the body." So we may say that the kinetic energy of a body is found by multiplying the mass of the body by the square of the velocity, and taking half the product.

132. Conservation of Energy. A body raised above the surface of the earth has potential energy, or energy of position; but when the body falls, it will perform just as much work as was necessary to raise it. In the act of falling, the body's potential energy will be changed into kinetic energy, or energy of motion; consequently what the body loses in potential energy, it gains in kinetic. On the instant of striking the ground, all the body's potential energy has been changed into kinetic energy.

When the body was falling, the total amount of energy it possessed, considering both potential and kinetic, was no more, no less, but always the same. It is true that the *form* of the energy was always changing, a greater and greater quantity becoming kinetic at the expense of the potential; the sum total, however, of all the energy possessed by the body remained constant. This doctrine of the preservation or indestructibility of energy, usually called the *conservation of energy*, is of the greatest importance in physics. Just as water may exist in any one of the forms, solid, liquid, or gaseous, and yet remain water, just so may energy exist in various forms and still

remain energy. Heat, light, and sound are forms of energy. When the falling body, about which we have been speaking, strikes the ground, there is a change of form of its kinetic energy: a part appears as heat, causing an increase in the temperature of the body; another part produces vibrations of the air, which reach the ear as sound, and if the body is of sufficient hardness, a flash of light may be seen. The body may, too, throw up a little earth when it strikes the ground, and also produce tremors and vibrations in the ground where it falls, so the kinetic energy which the body had in striking the ground has been changed from the kinetic form into that of heat, sound, etc.; but the sum of the amounts of energy in each of the separate forms is equal to that in the kinetic form.

Though its form may change, energy itself is indestructible; its amount in the universe is always the same.

THERMODYNAMICS.

133. Mechanical Equivalent of Heat. Whenever one body is rubbed by another, heat results; also whenever a piece of metal is hammered, the metal grows hot. In each of these cases mechanical action has been expended, and heat has been a result. In brief, then, heat has been a result of work. We have already become acquainted with a unit, the *calorie*, with which to measure quantities of heat, and also with the *gram-centimeter*, with which to measure work. It will be the object of the present article to give a concise account of the method of finding the relation between the unit of heat and the unit of work; in other words, of finding how many gram-centimeters are equivalent to one calorie.

The first precise numerical determination of this relation was made by Joule, in the following manner:

An upright cylindrical vessel is filled with water of a known temperature. A vertical shaft stands in the vessel. Extending from the sides of this shaft, in a horizontal direction, are paddles, which, when the shaft is turned, pass between stationary vanes projecting from the sides of the vessel. The paddles and the vanes keep the water thoroughly stirred when the shaft is turned. By means of an arrangement of wheels and a flexible string, the shaft is kept turning by a heavy body of known weight attached to the end of the string, and allowed to descend like a clock-weight.

As time goes on, and the water continues to be stirred, the temperature of the water rises. At the end of the experiment, the number of degrees through which the temperature of the water has risen is noted; also the distance through which the heavy body has fallen. After making necessary corrections for friction, the weight of the heavy body multiplied by the distance through which it has descended gives the amount of work expended in raising the temperature of the water. After making necessary corrections for the loss of heat from the water, the weight of the water multiplied by the number of degrees through which its temperature has been raised gives the number of units of heat received by the water during the process of stirring. By repeated experiment it has been found that *one calorie* is equivalent to *42,730 gram-centimeters.*

Definition. *The mechanical equivalent of heat denotes the amount of mechanical energy which must be transmuted into heat in order to yield one unit of heat.*

134. The Steam Engine. Mechanical energy can be transformed into heat. The process can be reversed; that is, heat can be transformed into mechanical energy. The steam engine is a contrivance for transforming heat into mechanical energy. Coal is burned on the grate beneath a boiler containing water. A large amount of the heat is wasted, but some of it vaporizes the water, turning it into steam. This steam is conveyed to a cylinder, and is alternately admitted automatically above and below a piston which is pushed back and forth by the expansion of the steam. A rod attached to the piston communicates its motion to a wheel which is kept revolving as long as the piston continues to move. By means of a belt, passing around this wheel, machinery can be kept in motion.

EXAMPLES.

1. Two men carry a weight of 152 lbs. between them on a pole, resting on one shoulder of each; the weight is three times as far from one man as from the other. Find how many pounds each supports, the weight of the pole being disregarded.

2. A man supports two weights slung on the ends of a stick 80^{cm} long placed across his shoulder. If one weight be two-thirds of the other, find the point of support, the weight of the stick being disregarded.

3. A man carries a bundle at the end of a stick over his shoulder. As the portion of the stick between his shoulder and his hand is shortened, show that the pressure on his shoulder is increased. Does this change alter his pressure upon the ground?

4. If the forces at the ends of the arms of a horizontal lever be 8 lbs. and 7 lbs., and the arms 8 in. and 9 in. long respectively, find at what point a force of 1 lb. must be applied at right angles to the lever to keep it at rest.

5. A uniform rod 3 ft. long and weighing 4 lbs. has a weight of 2 lbs. placed at one end. Find the center of gravity of the system.

6. A heavy beam is made up of two uniform cylinders whose lengths are as 3 is to 2, and whose weights are as 3 is to 5. Determine the position of the center of gravity.

7. A door 10 ft. tall, 5 ft. wide, and weighing 100 lbs. is attached to a wall by one edge in an upright position at two points, the first 1 ft. above the bottom, the other 1 ft. below the top. How great is the total downward pull upon the wall at the two supporting points? How great is the horizontal force exerted upon the wall at each point, and is this force a push or a pull?

8. A uniform bar 10 ft. long leans with one end against a vertical wall at an angle of 45°; the other end rests upon the ground. The bar weighs 20 lbs. There is no friction between the bar and the wall, so that the force there exerted is entirely horizontal. How great is this force?

9. A cord is fastened at each end, and a weight is suspended from it at a certain point, where the cord bends at a right angle. The pull exerted at one end of the cord is 3 pounds, and at the other end 4 pounds. How heavy is the suspended weight? (The weight of the cord is neglected.)

10. A balance has arms of unequal length, but the beam assumes the horizontal position when both scale-pans are empty. Show that if the two apparent weights of a body are observed when it is placed first in one pan and then in the other, the true weight will be found by multiplying these together and taking the square root.

Solution. Let a and b denote the lengths of the arms of the balance, and x the true weight of the body. If a weight, w_1, must be put into the pan attached to the arm whose length is a when the body is put into the other to produce equilibrium, we have, by the principle of moments,

$$bx = aw_1. \qquad (1)$$

If, when the body is transferred to the other pan, a weight, w_2, must be put into the pan which before contained the body to produce equilibrium, we have

$$ax = bw_2. \qquad (2)$$

From (1) $$\frac{a}{b} = \frac{x}{w_1},$$

From (2) $$\frac{a}{b} = \frac{w_2}{x},$$

$$\therefore \frac{x}{w_1} = \frac{w_2}{x},$$

$$x^2 = w_1\, w_2$$

$$\therefore x = \sqrt{w_1\, w_2}.$$

Hence, the true weight is equal to the square root of the product of the apparent weights.

11. The arms of a balance are in the ratio of 19 to 20; the pan in which the weights are placed is suspended from the longer arm. Find the real weight of the body which apparently weighs 38 lbs.

12. Two weights of 2 lbs. and 5 lbs. balance on a uniform heavy lever, the arms being in the ratio of 2 to 1. Find the weight of the lever.

13. In a hydraulic (or hydrostatic) press the area of the small piston face is 1 square inch and that of the large piston face is 50 square inches.

(*a*) If a force of 50 pounds is applied to the small piston, how great is the force exerted upon the large piston, provided there be no friction?

(*b*) How much *work* is done upon the small piston while it moves 6 inches, and how much is done at the same time upon the large piston? Name the unit in which the work is reckoned.

14. A trap-door of width 4 ft. and weight 8 lbs. has a load of 16 lbs. applied to the edge remote from the hinges. How many foot-pounds of work will be done in raising the trap-door till the edge remote from the hinges is 2 ft. above the floor?

15. A uniform plate of metal 10^{cm} square has a hole 3^{cm} square cut out of it, the center of the hole being 2.5^{cm} distant from the center of the plate. Find the position of the center of gravity of the plate.

16. A ladder, 30 ft. long and weighing 48 lbs., rests against a smooth wall, with its foot 15 ft. from the bottom of the wall. Find the pressure on the wall and on the ground, taking the center of gravity of the ladder as one-third of its length up.

17. A stone is thrown vertically upwards with a velocity of 192 ft. a second. Find how high it ascends, and how long it takes before returning to the hand.

CHAPTER VII.

MAGNETISM.

135. Magnets. The object of the experiments in this article will be to make clear the nature and the properties of magnets.

Experiment 114. *To find what happens on touching a magnet to different kinds of substances.*

Apparatus. A bar magnet; bits of wood, paper, glass, and also bits of iron and copper wire, or tacks.

Directions. Touch one end of the magnet to the various substances, one after another, with which you have provided yourself.

What happens in each case?

The effect which you observe on touching the magnet to a bit of iron is called *attraction*, and we say that the magnet *attracts* the bit of iron.

Experiment 115. *To find whether the magnet will attract a bit of iron without actually touching it.*

Apparatus. A bar magnet and a bit of iron.

Directions. Make the end of the magnet approach the iron very slowly.

Does the iron move before the magnet actually touches it?

Does the magnet exert its influence through the air?

Experiment 116. *To find whether the magnet will attract a bit of iron when the end of the magnet is covered with a piece of paper.*

Apparatus. A bar magnet; a bit of iron; paper.

Directions. Wrap one end of the magnet in a piece of paper.

Will the end thus wrapped attract the iron?

Experiment 117. *To find whether the attraction of the magnet in the preceding experiment acted through the paper or around it.*

Apparatus. The same as in the preceding experiment.

Directions. Wrap the whole magnet in paper.

Will the magnet now attract the iron?

Does the magnet exert its influence through the paper?

POLES.

136. North-Pointing Pole; South-Pointing Pole. In the next four experiments the *poles*, or parts of the magnet where the attraction is strongest, will be found, as well as the direction which the magnet will take when suspended so that it can be free to turn.

Experiment 118. *To find whether the magnet will attract at the center of its length.*

Apparatus. A bar magnet; an iron tack.

Directions. Touch the central portion of the length of the magnet to the tack.

Does the central portion of the magnet attract?

Experiment 119. *To find at what parts of a magnet the attraction is the strongest.*

Apparatus. A bar magnet; iron filings, fine and clean.

Directions. Lay the magnet on a sheet of paper. Sprinkle iron filings over the magnet; then grasping the magnet in the middle, carefully raise it from the paper.

What portions of the magnet attract the strongest?

How does this experiment enable you to tell?

How many poles has the magnet with which you have experimented?

Experiment 120. *To find whether one end of the magnet will point north.*

Apparatus. A bar magnet; a piece of fine silk thread.

Directions. Tie one end of a piece of thread about 1 ft. long round the middle of the magnet. Tie the free end of the thread to some support that will allow the magnet to hang freely. The support must not be in the neighborhood of any iron. Now adjust the loop round the magnet so as to make the magnet hang in a horizontal position. When the magnet is first hung up, the thread may twist a little and set the magnet spinning. If the magnet spins round two or three times, stop its motion with the hand, or else the thread will become twisted the other way, and, after a time, set the magnet spinning in the opposite direction, and thus cause loss of time.

When the magnet comes to rest, does one end point towards the north?

If so, mark this end with a piece of chalk. Hang the magnet up in a different part of the room, but not in the neighborhood of any iron.

In each part of the room where you hang the magnet up, does the marked end point north?

Note. In trying experiments with magnets, see that there are no pieces of iron or other magnets, in the neighborhood of the place where you are making the experiments; iron weights, retort-stands, keys, knives, etc., must be removed.

Experiment 121. *To find whether one end of another magnet will point north.*

Apparatus. Another bar magnet; a piece of fine silk thread.

Directions. Hang up the new magnet and repeat with it Exp. 120.

What is the result?

If one end points north, mark this end with chalk.

In a magnet, the pole that points north is called the *north-pointing* pole; the pole that points south is called the *south-pointing* pole. We shall write N-pointing pole for north-pointing pole, and S-pointing pole for south-pointing pole.

ATTRACTION AND REPULSION.

137. Attraction and Repulsion of Magnetic Poles. We shall now try some experiments for the purpose of getting a rule for predicting whether there will be attractions or repulsions when like poles of two magnets are brought near each other; also when unlike poles are brought near each other.

Experiment 122. *To find whether an N-pointing pole will attract an N-pointing pole.*

Apparatus. Two bar magnets.

Directions. Mark the N-pointing pole of each magnet with chalk. Hang up one of the magnets as in Exp. 121. When the magnet has come to rest, bring near its N-pointing pole the N-pointing pole of the other magnet.

What takes place?

Experiment 123. *To find whether an S-pointing pole will attract an S-pointing pole.*

Apparatus. The same as before.

Directions. Repeat Exp. 122, making use of the S-pointing poles instead of the N-pointing poles.

What takes place?

Experiment 124. *To find whether an N-pointing pole will attract an S-pointing pole.*

Apparatus. The same as before.

Directions. Repeat Exp. 122, but bring an N-pointing pole near an S-pointing pole.

What is the result?

From the results of Exps. 122, 123, and 124, state a rule for predicting what will take place when a pole of one magnet is brought near the a pole of another magnet.

138. The Compass. In the construction of the compass advantage has been taken of the fact that a bar magnet, when suspended in a horizontal position, points towards the north. A thin piece of steel that has been hardened and magnetized is poised upon a fine point of hard steel, such as the point of a needle. The thin piece of magnetized steel, called a *magnetic needle*, will move freely round on this point, but if left to itself, will always

come to rest with its poles pointing nearly north and south. The magnetic needle is indispensable in navigation, as thereby it is possible to ascertain the direction of the north pole of the earth, and, consequently, to direct a vessel's course by the chart.

METHOD OF MAKING A MAGNET.

139. Natural Magnets; Artificial Magnets. Besides the artificial magnets which you have been using, there are *lodestones*, or natural magnets, which have the same properties that you have already observed in magnets. The lodestone is an ore of iron, black and hard. This ore of iron is called *magnetite*, and is found in Asia Minor, Sweden, Arkansas, and in other parts of the world. The purpose of the next experiment is to make a magnet.

Experiment 125. *To find whether a magnet can be made from a piece of steel.*

Apparatus. A piece of watch spring about 8^{cm} long; a bar magnet; a compass.

Directions. Heat the bit of watch spring in a flame till it is just red hot. Take the spring from the flame and let it cool. You will now be able to straighten the spring. Heat the spring once more in the flame, and when it is red hot, plunge it quickly into cold water. The bit of steel is now hard and very brittle.

Stroke the bit of steel lengthwise with one pole of a strong magnet. Always make the strokes from one end of the steel, but never make back strokes. From time to time turn the steel so as to stroke the other side. By

using the compass, find which end of the little magnet you have made is the N-pointing pole. Keep this magnet for the next experiment.

Experiment 126. *To find, if a magnet be broken, whether the parts into which it is broken are magnets.*

Apparatus. The little magnet you made in the last experiment.

Directions. Break the magnet in the middle.

Is each part a magnet?

By breaking the magnet thus, have we succeeded in separating the poles so that there is an N-pointing pole, and an N-pointing pole only, on one of the pieces, and an S-pointing pole, and an S-pointing pole only, on the other piece?

Carefully considering what you have learned about magnets from the experiments you have performed, give as good a definition as you can of a magnet.

MAGNETIC INDUCTION.

140. Induced Magnetization. Whenever a magnet by its action produces magnetization in a piece of metal, this magnetization is called *induced magnetization.* The following experiment will make clearer the meaning of the term *induced magnetization.*

Experiment 127. *To find what is meant by magnetic induction.*

Apparatus. A bar magnet; a short cylinder of soft iron; a small iron tack.

Directions. Touch the cylinder of soft iron to the tack.

Does the iron attract the tack?

Now put one end of the soft iron cylinder against one end of the magnet, and holding the two together, touch the other end of the soft iron cylinder to the tack.

Is the tack attracted?

Remove the magnet from the soft iron.

What does the tack do?

Put a piece of paper between the end of the magnet and the cylinder of soft iron.

Will the soft iron now attract the tack?

Does the piece of soft iron become a magnet while the bar magnet is near it?

If the soft iron does become a magnet, can you say it becomes a magnet by induction?

MAGNETIC CURVES.

141. Curves formed by Iron Filings about a Magnet. Whenever iron filings are sprinkled upon a magnet, the filings arrange themselves in clusters at the poles of the magnet, and if the magnet is lying on a table, the filings, which fall upon the table in the neighborhood of the magnet, unite in lines and assume the form of curves.

Experiment 128. *To find the general form of the curves of iron filings sprinkled near a magnet.*

Apparatus. A bar magnet; iron filings; a muslin bag; a sheet of paper 20^{cm} square.

Directions. Lay the bar magnet on the table, and over it lay the sheet of paper, which should be supported by

pieces of wood so as to make the surface *level* (meter sticks are good for this purpose). Holding the muslin bag filled with iron filings about a foot above the paper, sift the iron filings over the paper. Tap the paper lightly till the filings set themselves along lines. These lines are called *magnetic curves.* Avoid getting too great a quantity of filings on the paper. Sketch in your note-book the outline of the magnet and the magnetic curves.

Definition. *The space through which the magnetic influence of a magnet extends is called the magnetic field of the magnet.*

142. Lines of Magnetic Force. The preceding experiment helps us to see what is meant by the term *magnetic field.* "This expression merely denotes the *space all round a magnet through which it is capable of exerting an influence upon soft iron or upon other magnets.* The magnetic curves by which the magnetic field may be mapped out represent, in the first place, ropes or chains more or less continuous, into which the iron filings arrange themselves when they are rendered free to turn by the influence of tapping." The iron filings, in fact, become little magnets by induction. Had we used, instead of iron filings, a series of very small magnetic needles free to move, these would have similarly arranged themselves along the magnetic curves. These magnetic curves are called *lines of magnetic force.*

Definition. *A line of magnetic force is a line or path in a magnetic field, such that if we walk along it with a little magnetic needle suspended from our hand, this needle will always point along the path.*

The earth is a huge magnet having its *magnetic poles* comparatively near its *geographical poles*, but not coincident with them. The lines of magnetic force of the earth's magnetic field curve round over the surface of the earth from pole to pole and act on a freely suspended magnet, that is, a magnet that can turn to the right or left, also up or down, in such a manner that the magnet turns till it lies as nearly as possible along one of these lines.

Using a magnetic needle as suggested in the definition of a line of magnetic force, we shall now take up some experiments on tracing lines of magnetic force.

Experiment 129. *To find the shape of the lines of magnetic force about a bar magnet whose N-pointing pole is turned towards the north.*

Apparatus. A bar magnet; a small compass; a sheet of paper about 50^{cm} square; seven copper tacks.

Directions. Fasten the sheet of paper on the table with copper tacks, not with iron tacks, at each corner. Remove all iron from the neigborhood. (Why?) Draw a line extending across the paper and pointing north. This is done by laying a meter stick on the paper, placing the compass on the stick, and turning the stick till the compass needle is parallel with the sides of the meter stick. Then with a pencil draw a line beside the meter stick. Lay the bar magnet on the middle portion of this line with the N-pointing pole pointing north. Fasten the magnet in place by sticking two tacks into the table on one side, and a third tack on the opposite side, as shown in Fig. 108. Mark the outline of the magnet on the

paper. Place the compass at point 1, Fig. 108, of the magnet, and then move it away in the exact direction in which the compass needle points. A good way to do this is to make a dot on the paper just at the end of the compass opposite the end of the needle that is turned away from the N-pointing pole of the magnet. Then move the compass till the end of the needle that is next to the magnet is over this dot. Make a dot at

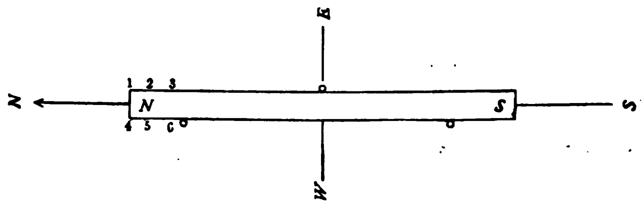


FIG. 108.

the other end of the needle on the paper, and continue this process till the path reaches the edge of the paper or returns to the magnet.

Trace upon the paper the lines thus followed by the compass, putting arrowheads at several points to indicate the direction in which the N-pointing pole of the compass needle points at these places. Then place the compass a little nearer the middle of the magnet (at 2), and starting anew trace another line and mark it as before. Then, beginning still farther (at 3) toward the middle of the magnet, do as before. Finally, start not more than 3^{cm} or 4^{cm} from the middle of the magnet and trace a line. Trace an equal number of lines on the western side of the magnet. There should be traced in all eight lines. Paste the paper into your note-book.

Experiment 130. *To find the shape of the lines of magnetic force about a bar magnet whose N-pointing pole is turned towards the south.*

Apparatus. The same as in the last experiment, with a fresh sheet of paper.

Directions. Fasten the sheet of paper to the table. Draw a line across it towards the north. Put the magnet on the line and fasten the magnet with its N-pointing pole pointing south. Draw an outline of the magnet on the paper. Then use the compass and trace lines as before, marking them all with arrowheads to indicate the direction in which the *N-pointing* pole of the compass needle points. Start from the same points of the magnet as in the last experiment. Draw eight lines in all. Paste the paper into your note-book.

Experiment 131. *To find the shape of the lines of magnetic force about two bar magnets placed side by side, parallel, and with their N-pointing poles pointing north.*

Apparatus. The same as in the last experiment, with an additional magnet and a fresh sheet of paper and three more copper tacks.

Directions. Fasten the paper to the table; draw the line towards the north; lay the two magnets parallel on the paper (Fig. 109), the N-pointing poles pointing north and about 15^{cm} apart, measured on an east and west line. Trace the outline of each magnet. Proceed to trace the lines of force as in the experiments already performed. Start from the positions indicated by the figures in the diagram. Trace the lines followed by the compass needle, and paste the paper into your note-book. When you are

tracing the lines of force in the space between the magnets, be careful not to move the compass from the field of one magnet into that of the other.

Examine the three diagrams which you have made in

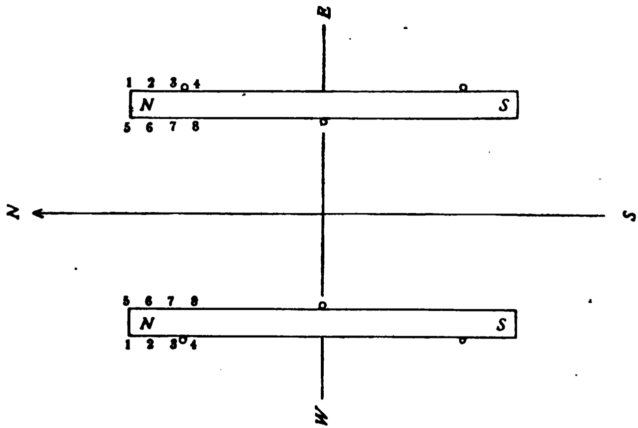


FIG. 109.

Exps. 129, 130, and 131, and then answer the following questions:

Is there any case in which *all* the lines of force return to the magnet?

Is there any case in which *none* of the lines of force return to the magnet?

Is there any case in which *some*, but not all, of the lines of force return to the magnet?

Can you account for the shape of the lines of force?

SUGGESTION. Keep in mind that for certain positions of the magnet the magnet's action and the earth's action upon the compass needle oppose each other, while for other positions these actions assist each other.

In the case where you had the two magnets side by side, were the curves between the magnets crowded more closely together than the corresponding curves on the outside?

Did you find in the case of the two magnets a neutral region, that is, a place where the compass needle could, by a very slight variation of its position, be made to point in an entirely new direction?

143. Theory of Magnetism. Keeping in mind what our experiments in magnetism have taught, let us try to frame a theory of magnetism. The fact that lines of force leave an ordinary bar magnet at its ends, and that they also return to its ends, might lead us at first thought to conclude that magnetism exists only at the ends of a magnet; but when we call to mind the fact that a magnet may be broken into many pieces which have each an N-pointing pole and an S-pointing pole of equal strength, we infer that, could we continue this subdivision of the magnet till its ultimate particles, or molecules, were reached, each particle would be found to be a magnet with an N-pointing pole and an S-pointing pole of equal strength. When to make a magnet of a bar of steel we stroke it with a magnet, we find that the magnet loses none of its magnetism (it can lift as heavy a weight after we have stroked the steel as before); so we look upon a piece of unmagnetized steel as consisting of particles, each a magnet in itself, but arranged in such a way as to neutralize each other; and we look upon the process of magnetization as consisting in a rearrangement of the particles, so that all the N-pointing poles shall be turned

in one direction, and all the S-pointing poles in the opposite direction. In a bar magnet, then, the layers of particles forming the ends would be the only particles the magnetism of which is not completely neutralized by their neighbors. The outer face of the layer of particles at one end of the bar would have free N-pointing magnetism; the outer face of the layer at the other end, free S-pointing magnetism.

144. Coercive Force; Residual Magnetism. A piece of hardened steel is more difficult to magnetize than a piece of soft iron; but, on the other hand, the piece of steel retains its magnetism, while the soft iron quickly loses all but a trace. These facts are explained by saying that the particles composing the steel bar are difficult to turn out of the positions which they have when the bar is in its ordinary unmagnetized state; but after the bar has been magnetized, it is difficult for them to return into their old positions, consequently, the bar of steel remains magnetized. In the case of a bar of soft iron, however, the particles composing it are easily turned out of their positions; but after the source of magnetization is removed, these particles readily return very nearly to their former positions, and all but a trace of magnetism disappears. The power of resisting magnetization or demagnetization is called *coercive force*, or better, *retentivity*. Thus, hardened steel has great retentivity; soft iron, little.

The trace of magnetism that remains in a piece of soft iron, after its *temporary magnetism* has disappeared, is called *residual magnetism*.

CHAPTER VIII.

ELECTRICITY.

145. Effects due to Electricity. We shall begin our study of the subject of electricity with a few simple experiments for the purpose of observing some of the effects due to electricity. To produce the electricity, we shall use a Bunsen cell. The Bunsen cell is a contrivance for obtaining electricity; it consists of the following parts:

(1) A glass jar (Fig. 110, 1), containing dilute sulphuric acid[1] to a depth of a few inches.

(2) A hollow cylinder of zinc (Fig. 110, 2), to be placed in the glass jar.

(3) A porous cup (Fig. 110, 3) of unglazed earthenware, containing a mixture of sulphuric acid, water, and bichromate of potash.[2]

(4) A prism of carbon (Fig. 110, 4), to be placed in the porous cup.

(5) Two brass clamps, not shown in the figure. One of these clamps is fastened to the zinc cylinder, the other to the end of the carbon prism.

The porous cup containing the liquid and the carbon is placed within the hollow of the zinc cylinder contained in the glass jar. Fig. 110, 5, represents a section of the cell when all the parts have been brought together and

[1] Consisting of one part by weight of sulphuric acid and twelve parts of water.

[2] Consisting of eight parts by weight of water, two parts of sulphuric acid, and one part of bichromate of potash.

arranged in the proper manner. The different parts shown in the section of the cell can be easily distinguished by noting that the first, the upper, arrow touches the glass jar, the second arrow touches the zinc, the third arrow touches the porous cup, and the fourth arrow touches

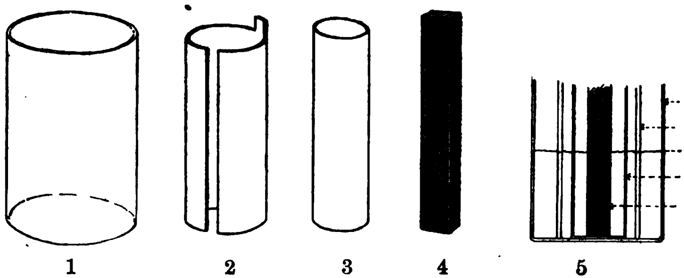


FIG. 110.

the carbon prism. The horizontal line marks the surfaces of the liquids, which should be at the same level in both the jar and the porous cup. Be careful not to spill any of the acid upon the clothing or upon the table.

Experiment 132. *To find whether electricity produces a physiological effect.*

Apparatus. A Bunsen cell; two copper wires, each about 40^{cm} in length, with ends brightly polished with sandpaper.

Directions. Attach one end of one wire to the clamp fastened to the zinc; there is a little hole in the top of the clamp into which the end may be slipped and then firmly fastened in place by means of a screw. In the same manner attach the end of the other wire to the clamp which is fastened to the carbon.

Touch the free end of one of the wires to the tip of the tongue.

What is the sensation?

Touch the end of the other wire to the tip of the tongue, after removing the first wire from the tongue.

What is the sensation?

Touch both wires to the tongue, so that the ends of the wires are about 1^{cm} apart.

What is the sensation?

Which wire seems to cause the sensation?

The sensation you have felt is called a "physiological effect of the electric current." It is customary to speak of electricity as flowing, and this flow of electricity is called an *electric current.* Furthermore, the current is spoken of as flowing *from* the carbon, through the wire joining the carbon, *to* the zinc; when the current reaches the zinc, it flows through the liquids in the cell back to the carbon again. This process goes on till the cell becomes exhausted, by reason of some of the materials which compose it becoming used up, or till the wire is disconnected from either the zinc or the carbon. In the experiment you have just performed the current flowed through the tip of the tongue from one wire to the other.

Experiment 133. *To find whether electricity can produce light.*

Apparatus. Two Bunsen cells; three pieces of copper wire, each about 40^{cm} long; a file.

Directions. By means of one of the wires join the zinc of one cell to the carbon of the other. Join an end of one of the remaining pieces of wire to the free clamp of

one cell, and an end of the other wire to the free clamp of the other cell. To the end of either of the two wires last mentioned attach the file, and rub the end of the other wire along the file.

What do you see?

NOTE. An arrangement of two or more cells is called a battery; just as in military science a battery of guns means an arrangement of two or more guns.

Experiment 134. *To find whether the electric current can produce heat.*

Apparatus. A Bunsen cell; a piece of iron wire, No. 30 B. & S.; a piece of copper wire of the same size as the iron.

Directions. Twist one end of the iron wire round the end of one of the copper wires leading from the cell; then move the end of the other copper wire leading from the cell along the iron wire.

Does the iron wire grow warm?

In place of the iron wire use the piece of copper wire.

Does the copper wire grow as warm as the iron wire?

In the experiments on heat, which did you find to be the better conductor of heat, iron or copper?

If a poor conductor of electricity is drawn out into a wire it will be heated a good deal.

Hence, which is the better conductor of electricity, iron or copper?

Experiment 135. *To find what effect is produced on a magnet by an electric current in its neighborhood.*

Apparatus. A Bunsen cell; a small compass.

Directions. Join the ends of the wires leading from the cell by twisting them together. Bring the loop of

wire thus formed close to the face of a compass. What happens to the compass needle?

Does the electric current act in any way like a magnet?

146. The Measurement of the Strength of an Electric Current. In order to measure the strength of an electric current, you might make use of any one of the effects you have studied in Exps. 132, 133, 134, and 135, thus:

(1) The greater the physiological effect, the stronger the current.

(2) The greater the luminous effect, the stronger the current.

(3) The greater the heating effect, the stronger the current.

(4) The greater the magnetic effect, the stronger the current.

For the purpose of measuring the strength of a current of electricity it has been found best to make use of its magnetic effect. Hence we shall make a careful study of the magnetic effect of the electric current.

ACTION OF CURRENTS ON MAGNETS.

147. Magnetic Field produced by an Electric Current. Whenever a current of electricity flows through a conductor, a magnetic field is produced, by the current, in the neighborhood of the conductor. It will be our purpose to find out something about the lines of force in a magnetic field which is produced in this way.

Experiment 136. *To find the shape of the lines of force when a magnetic field is produced in the neighorhood of a conductor by a current of electricity flowing through.*

Apparatus. Two Bunsen cells; a stout copper wire; a piece of cardboard about 20^{cm} square; fine iron filings; pieces of copper wire for connecting the cells.

Directions. Join the zinc of one cell to the carbon of the other by a piece of wire. Pierce the cardboard at its center with a hole large enough for the stout wire to pass through. Place the cardboard in a horizontal position with the stout wire protruding vertically through the hole both above and below. To one end of this stout wire join, by means of a piece of wire, the free carbon, and to the other end of the stout wire, also by means of a piece of wire, join the free zinc. A current is now flowing through the stout copper wire. Sprinkle on the cardboard a few iron filings, and gently tap the cardboard.

In what form do the filings arrange themselves on the cardboard?

What is the form of the lines of force in the magnetic field surrounding the conductor?

Experiment 137. *To find the relation between the direction of the electric current and the direction in which a compass needle points, when placed in the neighborhood of the current.*

Apparatus. A Bunsen cell; a compass; a piece of copper wire 2^m or 3^m long; two connecting cups; five copper tacks.

Directions. Bend the piece of wire into the form of a square (Fig. 111), and have the ends of the wire project

5^{cm} or 10^{cm} from the corner of the square. Place this square on the table so that two sides shall run east and west, and the other two north and south. You can make sure that the square is placed correctly by means of a compass, adjusting the square till the sides that are to run north and south are parallel to the needle. Join (Fig. 111), by means of connecting cups, the ends of the wire forming the square to the ends of the wires from the cell. Be careful to scrape the insulating material from the ends of the wires, in order to get good contact. Fasten the corners of the square to the table with copper tacks. The figure represents the arrangement. The arrowheads denote the direction in which the current flows in going from the carbon of the cell through the wire back to the zinc of the cell. Call the side of the square through which the current, on its way from the carbon to the zinc, flows east, *E*, that in which it flows west, *W*, etc.

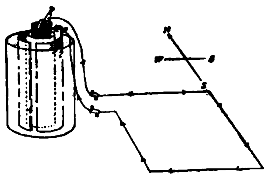


FIG. 111.

Draw in your note-book a diagram similar to the figure. Now place the compass on the wire at the middle of the side *E*, and indicate, by drawing a straight line through the middle point of the line *E* in your note-book, the general direction in which the N-pointing pole of the compass needle turns. At one end of this line put an arrowhead, to show which end represents the N-pointing pole of the needle.

Now draw another diagram in your note-book just like the one you have already drawn. Lift the wire a little way on the side *E*, without detaching it from its fasten-

ings, and slip the compass underneath the middle of the side *E*. In the new diagram, by drawing a line as before, indicate the direction in which the needle points. Do the same thing with each of the sides *S*, *W*, and *N*, marking in one diagram the position of the needle when placed above the middle point of each side, and in the other diagram the position of the needle when the compass is placed beneath the middle point of each side.

QUESTIONS.

By referring to your diagrams, answer the following questions:

1. A wire is stretched north and south, and a current runs from south to north in the wire; if a compass is placed above the wire, will the N-pointing pole of the needle be on the east or on the west side of the wire?

2. Suppose the conditions mentioned in 1 remain the same in all respects, except that the compass is placed under the wire; on which side (east or west) will the N-pointing pole of the needle lie?

3. Suppose all the conditions mentioned in 1 remain unchanged, except that the current flows from north to south; what will be the position of the N-pointing pole of the needle?

4. If in the last question the compass had been placed under the wire, everything else remaining the same as before, what position would the N-pointing pole of the needle have taken?

5. In a wire stretched east and west a strong current of electricity flows from east to west; if a compass is placed above the wire, will the N-pointing pole turn to the north or to the south side of the wire?

6. Suppose all the conditions of 5 remain unchanged, except that the compass is placed underneath the wire; what position does the N-pointing pole of the needle take?

7. Suppose all the conditions mentioned in 5 remain unchanged, except that the current flows from west to east; in what position will the N-pointing pole of the compass needle point?

8. Suppose all the conditions mentioned in 7 remain unchanged, except that the compass is placed below the wire; what position will the N-pointing pole of the needle now take?

148. Rule for Direction of Deflection of Magnetic Needle by Electric Current. A line of force is said to have the direction in which the N-pointing pole of a compass needle points, when placed on the line of force; bearing this in mind, answer the following question:

If you look along the wire in the direction in which the current flows, will the lines of force run round the wire in the same direction as the hands of a clock move?

Your answer to this question gives the rule for finding the direction of deflection of a magnetic needle by an electric current.

LINES OF FORCE ABOUT A COIL.

Experiment 138. *To find the positions of the lines of force surrounding a conductor which consists of a coil of wire.*

Apparatus. A wooden ring, round which is wound 15 turns of insulated copper wire, mounted on a base (this wire is wound with cotton or silk to keep the electricity from going across from one spire to the next; a wire thus covered is called *insulated wire*); a Bunsen cell; a compass; a meter stick.

Directions. To each of the outer binding posts join a wire from the cell (Fig. 112). By counting the coils of wire you can assure yourself that there are 15 turns on the wooden ring, and by carefully examining the apparatus you will see that there are 15 turns of wire "in circuit," which is a brief form of saying that all these 15 turns of wire are traversed by the current. Place the ring so that the *plane* of its coil shall be east and west, and so that the current shall flow from east to west through the wires on the top of the circle. To find whether the current is flowing in

the way described, place the compass on the top of the circle, observe the position of the needle, and refer to the rule about the deflection of a magnetic needle which you were asked to state. On the cross-piece place a meter stick with its center on that of the cross-piece, and have the meter stick at right angles to the plane of the coil.

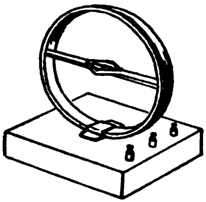


Fig. 112.

Support the ends of the meter stick by means of blocks to keep the stick steady. The stick should be pointing north and south. On the meter stick place the compass 10^{cm} south of the center of the coil, and record in a diagram, by means of a straight line, the direction in which the needle points. Now move the compass north along the meter stick 5^{cm}; record as before the position of the needle. Then move the compass 5^{cm} north to the center of the coil; record. Again move the compass 5^{cm} north; record. Finally, move the compass 5^{cm} north from the last stopping-place, and record the position of the needle.

Place the compass within the coil, but close to its eastern side. Record, by means of a diagram, the direction of the needle. Then place the compass in turn north, east, and south of this portion of the coil (and all the time close to it), recording the direction in which the needle points for each position of the compass. Carrying the compass round the western side of the coil, make and record similar observations. Fig. 113 may help to make the meaning of this paragraph clearer. The shade lines represent the part around which the compass is carried; the circles, the positions of the compass.

Now reverse the current in the coil. This is done by changing the connections with the cell, putting the wire attached to the zinc in the binding post that before held the wire from the carbon, and the wire from the carbon into the binding post that before held the wire from the zinc. Repeat all the operations described in the preceding part of this experiment, and make new diagrams.

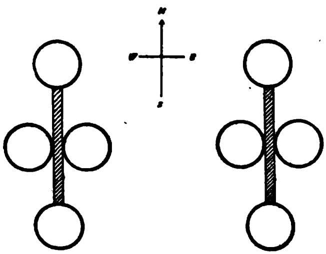


FIG. 113.

Compare the diagrams obtained by moving the compass round the east and west sides of the coil with those obtained in Exp. 131 in magnetism, when the two magnets were placed parallel to each other. In this comparison, point out the differences and the likenesses in the arrangement of the lines of force about the magnets and about the coil.

In this experiment, have you noticed anything in the course of your work that would make it appear as if the coil acted as a magnet, having one pole at one face of the coil and the other pole at the other face?

CHEMICAL ACTION IN THE CELL.

149. Study of the Action that takes Place in the Cell. As we have now studied some of the *effects* of electricity, we are in a better position to study the action that goes on in the cell. In beginning the study of the cell, we shall take the simplest form, not the complicated arrangement that we have been using.

Experiment 139. *To find what action dilute sulphuric acid has upon a strip of zinc and upon a strip of copper placed in it.*

Apparatus. A small glass tumbler; a strip of sheet zinc and a strip of sheet copper, each about 10^{cm} long and 1.5^{cm} wide, and each having about 50^{cm} of insulated copper wire (size about No. 20 B. & S.) soldered to one end.

Directions. At the sink, fill the tumbler to within 1^{cm} of its top with dilute sulphuric acid (1 part by volume of acid to 20 parts by volume of water). Do not let any of the acid fall upon the floor, table, or clothing. Put into the tumbler, close to one side, the strip of clean zinc, and close to the other side the similar strip of copper; each strip should rest on the bottom of the tumbler, and the upper end of each should be bent so as to clasp the edge of the tumbler, as shown in Fig. 114. Now observe for about a minute what happens at the surface of each strip, taking care that the strips do not touch each other.

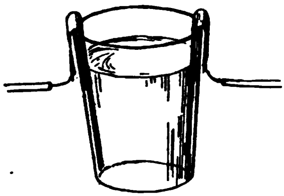


FIG. 114.

What happens at the surface of each strip?

Experiment 140. *To find what happens when a strip of zinc and a strip of copper, plunged into dilute sulphuric acid, are joined by a piece of copper wire.*

Apparatus. The same as in the preceding experiment; also a compass and a coil mounted on a base as shown in Fig. 112; mercury.

Directions. Having the tumbler, plates, and acid arranged as in the last experiment, put the two strips into metallic connection, by attaching their wires to the ends

of the 15-turn coil. Before the wires are attached, however, the compass should be placed on the center of the shelf running across the coil, and the coil turned round till the needle of the compass lies in the plane of the coil. The little instrument thus arranged, consisting of the compass and the coil, is called a *galvanoscope*. It enables us to detect the presence of an electric current; for, as we already know, if a current flows through the coil, the needle is deflected from its natural position; if the current is strong, the needle will be deflected a good deal; if the current is weak, the needle will be deflected only a little. *Whenever the galvanoscope is used, it must be placed in a position to make its coil lie in the magnetic meridian;* that is, the needle of the compass on the little shelf running through the center of the coil must lie in the plane of the coil. *After the galvanoscope has been placed in position, on no account must its position be changed till the experiment is completed.*

Observe for a short time what happens at the surface of each of the strips in the acid.

Do any bubbles form and rise? Do more rise from one strip than from the other?

Observe, too, the behavior of the galvanoscope needle.

After tapping the instrument lightly to make sure that the needle has not through friction on the pivot come to rest in the wrong position, record the number of degrees through which the tip of the needle has turned.

Disconnect the zinc strip, and plunge into mercury that part only which has been under the acid. Then, after wiping from the zinc any loose drops of mercury, replace it in the acid.

What happens now at the surface of the zinc?

Now connect, as before, with the galvanoscope.

What happens at the surface of the zinc?

Be careful that the two strips are as far apart and immersed to the same depth as before.

What position does the galvanoscope needle take?

What happens at the surface of the copper strip?

Experiment 141. *To find whether the strength of the current from the single-fluid cell changes as time goes on.*

Apparatus. The same as in the last experiment.

Directions. Wipe the plates dry and clean. Then put them into the liquid and *immediately* get the reading of the galvanoscope, that is, the number of degrees through which the N-pointing pole of the needle is deflected. Readings should then be taken every two minutes for a period of ten minutes. If at the end of ten minutes there are any bubbles on either of the strips or on both, rub off these bubbles with a bit of wood without removing the strips from the acid, *taking care to have no mercury come in contact with the copper.* Record the position of the needle in every case.

Finally, try the effect of amalgamating (covering with mercury) the copper; the zinc has been amalgamated.

How do you account for the position of the needle?

150. Discussion of the Chemical Action that takes Place in the Cell. Chemists teach that a molecule of sulphuric acid consists of two parts of hydrogen (the bubbles that you saw rising from the zinc before the circuit was closed were bubbles of hydrogen), one part of sulphur,

and four parts of oxygen. When this molecule of sulphuric acid comes in contact, under proper conditions, with zinc, chemical action takes place and the two parts of hydrogen in the molecule are pushed or crowded out by one part of zinc, and the hydrogen passes off in the form of bubbles, while the compound formed with the zinc remains behind; this compound is called *zinc sulphate*.

151. The Reason for Amalgamating the Zinc. If the zinc is acted upon by the acid when the battery is not in use, a waste of zinc takes place; pure zinc, when placed in sulphuric acid, is but little affected. Pure zinc, however, is expensive, so we use impure, but amalgamated, zinc. Impure zinc when amalgamated behaves in sulphuric acid much like pure zinc; the mercury dissolves the zinc, leaving the impurities unchanged, thus spreading a coating of pure zinc over the surface of the plate.

Under ordinary circumstances, sulphuric acid acts but slightly upon copper. Any action seen at the surface of the copper strip in Exp. 140, even when the circuit was closed, was due to hydrogen bubbles, set free by a chemical action that did not affect the copper.

152. Polarization of the Cell. The weakening of the current of a single-fluid cell after the circuit is closed, and the recovery of strength by the current when the plates are thoroughly rubbed, are phenomena that demand our attention. This weakening of the current is evidently not due to an exhaustion of the fluids in the cell, but rather to the condition produced at the surface of one or both of the metallic strips by the action of the cell. In fact, the weakening of the current is due to

the deposition of bubbles of hydrogen upon the plate of copper; for when the bubbles are removed, the strength of the current returns. This deposition of hydrogen bubbles upon the copper plate is called *polarization of the cell.* The coating of hydrogen bubbles upon the plate acts in two ways to weaken the current:

(1) Since the hydrogen is a poor conductor of electricity, it opposes the flow of the current;

(2) When the copper strip is covered with a film of hydrogen, we have practically a plate of hydrogen exposed to the action of the acid; the result is the starting of a current in the opposite direction, that tends to neutralize the first, which flows in the cell from the zinc to the copper. The more the copper plate gets covered with the hydrogen bubbles, the stronger does this neutralizing tendency become, until the first current is overpowered; then the galvanoscope indicates no current in either direction.

In order to avoid the formation of hydrogen bubbles upon the copper plate, use is made of the *two-fluid* cell. In this cell the copper plate is immersed in a liquid which does not allow hydrogen to reach the copper plate. Just how this is done will become clearer after performing the following experiment with a two-fluid cell.

Experiment 142. *To find what action goes on in the two-fluid cell.*

Apparatus. A large tumbler; a small porous cup that will sit easily into the tumbler; a piece of zinc 10^{cm} long, 2.5^{cm} wide, 0.4^{cm} thick, with a piece of copper wire 40^{cm} long soldered to it; a piece of sheet copper 10^{cm} square, with a copper wire 40^{cm} long like that of

the zinc; a galvanoscope; dilute sulphuric acid (one part by volume of acid to twenty parts by-volume of water); a saturated solution of sulphate of copper; mercury.

Directions. Put the zinc into the porous cup, and then fill this cup with diluted sulphuric acid to within 2^{cm} of its top. Put the cup containing the zinc and acid into the tumbler, and then pour into the tumbler sulphate of copper solution till this liquid stands as high in the tumbler as the acid stands in the porous cup. Then remove the zinc from the acid. The zinc is now in a condition to be amalgamated, which should be done by dipping it into the mercury. After amalgamation, wipe the zinc to remove loose drops of mercury (do this over an iron pan to save the mercury), and then weigh the amalgamated zinc to 0.1^{gm}. Wash, dry, and then weigh with equal accuracy the copper sheet. Bend the copper plate somewhat so that it may partly encircle the porous cup, and put the plate thus bent into the sulphate of copper in the tumbler. Replace the zinc in the porous cup. You already know that if one wire from a cell is joined to one of the outside posts of the galvanoscope and the other wire from the cell to the other outside binding-post, then the current from the cell will flow through 15 turns of the galvanoscope coil. On the other hand, if one of the wires from the cell is joined to the middle post and the other wire to one of the outside posts, the current from the cell will flow through 5 or 10 turns of the galvanoscope coil, according to which of the outer posts the wire is attached to. Join the cell to the galvanoscope so that the current shall flow through the 5-turn section. As soon as the needle comes to rest (before reading the instrument, it should be

lightly tapped so that the needle may disengage itself if caught by friction on the pivot), record its position, stating how many degrees it is deflected from the magnetic meridian, and also whether this deflection is towards the east or towards the west. Readings should be taken and recorded at 5-minute periods for half an hour. Then disconnect the cell, remove both the zinc and the copper, rinse each gently, taking care not to remove any deposit that may have formed; and then, without rubbing, dry the plates, and weigh them again.

What has been the gain or loss in weight for each plate?

From an inspection of the record of the galvanoscope readings, should you say that the current had become and remained constant soon after the beginning of the experiment?

At the beginning of the experiment the walls of the porous cup are often not thoroughly wet. After a little while, however, the liquids soak in and the walls become moist. The current flows more easily through a substance thoroughly wet than through the same substance when only moist.

NOTE. This two-fluid cell with which you have experimented is called, from the name of its inventor, the Daniell cell.

153. Chemical Action in the Two-Fluid Cell. Within the porous cup, in Exp. 142, the zinc pushes out the hydrogen in the sulphuric acid and takes its place. The hydrogen thus set free does not appear in the form of bubbles, but in the part of the cell outside the porous cup, it pushes out the copper of the copper sulphate

(copper sulphate consists of 1 part of copper, 1 of sulphur, and 4 of oxygen). The copper freed in this manner is deposited upon the copper plate. Instead of hydrogen being deposited upon the copper plate, copper is deposited and polarization is prevented. The cell gives a steady current till the materials (generally the copper sulphate) of the cell become exhausted. The Bunsen[1] cell is a two-fluid cell.

154. The Ampère. Exp. 142 gives a means of understanding clearly what is meant by the *ampère*, the unit of current strength.

Definition. *An ampère is a current of such strength as to set free from its chemical combinations 0.0003281 of a gram of copper in one second.*

The usual strength of current for an arc-lamp is about 10 ampères.

Turn back to your record of Exp. 142, and with the data there furnished, compute the weight in grams of copper set free (that is, deposited on the copper plate) from the sulphate of copper solution in one second by the current.

If a current of one ampère sets free in one second 0.0003281^{g} of copper from its chemical combinations, what was the average strength in ampères of the current passing through your Daniell cell during the half-hour test?

155. The Commutator; the Rheostat. In the next experiment you will have occasion to use two pieces

[1] It is better to call the cell you used in the first experiments in electricity the Poggendorff cell. The true Bunsen cell has nitric acid in the porous cup instead of the mixture of water, sulphuric acid, and bichromate of potash which you used.

of apparatus, a description of which is here given for convenience.

Hitherto, when you wished to change the direction of a current through a conductor (the galvanoscope coil, for instance), the ends of the wires from the cell were interchanged at the ends of the conductor. This changing of connections was troublesome and took time. To effect this change quickly and conveniently, a piece of apparatus called a *commutator* has been contrived.

The commutator (Fig. 115) consists of a block of wood in one side of which four holes, called *cups*, are made. There are four binding-posts screwed into the sides of the block, one entering each cup. The cups are partly filled with mercury. There are also two wires passing through a disc of wood and bent at the ends, by means of which the cups can be connected in pairs.

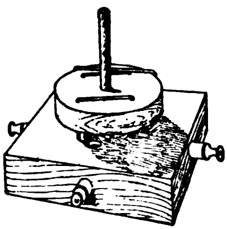


FIG. 115.

The cell must always be joined to a pair of opposite binding-posts, and the galvanoscope wires must be joined to the other pair of binding-posts.

If the set of movable wires, carried by the wooden disc, dip into the cups, the current flows from the cell through the galvanoscope. If the disc is raised, given a quarter turn and then lowered, the current will flow in the opposite direction through the galvanoscope.

The rheostat, shown in the upper part of Fig. 116, consists of a frame made by attaching an upright at each end of a piece of plank about a meter long, and joining the upper ends of these uprights by a meter stick parallel to

the plank. On one of the uprights are placed four horizontal rows of binding-posts. Each row consists of two posts. On the other upright, near its lower end, is placed a horizontal row of two binding-posts. Between the two uprights is stretched a piece of No. 30 German silver wire, held in place at its ends by the binding-posts which are nearest the plank and are placed nearest the inside edge of each upright. Another piece of No. 30 German silver wire is fastened to the outer binding-post of the lowest row. This wire is then carried round the outer edge of the upright and brought to the other upright round whose outer edge it is carried and fastened to the binding-post nearest this edge. Another piece of No. 30 German silver wire is fastened to one of the binding-posts in the next row above, is carried once round the supports, and fastened at its end to the other binding-post of the pair. The same thing is done for the next pair of posts, only a No. 28 German silver wire is used. Finally, round the upper part of the rack are wrapped 20^{m} of silk-covered, No. 30, copper wire; an end of this wire terminates at each of the upper pair of binding-posts.

ELECTRICAL RESISTANCE.

156. Electrical Resistance of Wires. When water runs through a pipe, the friction between the walls of the pipe and the flowing water resists the flow. The current of water is not so strong as it would be were the impeding effect of friction removed. The resistance to a flow of water in two pipes of the same diameter, but of unequal lengths, is greater in the longer pipe. On

the other hand, in two pipes of the same length, but of unequal diameters, the resistance is greater in the smaller.

Thus, just as a pipe through which water runs offers to the flow a resistance, which depends upon the size and length of a pipe, so a wire offers resistance to the flow of electricity. It will be the object of the next two experiments to find, if possible, some relation between the length of the wire and the amount of resistance (Exp. 143); also between the area of cross-section of the wire and the amount of resistance (Exp. 144).

Experiment 143. *To find what effect the length of a wire has upon its resistance to the flow of an electric current.*

Apparatus. A Daniell cell; a rheostat; a commutator; a galvanoscope; an "English" binding-post.

Directions. By means of a bit of copper wire connect the pair of binding-posts at the bottom of the left-hand upright (Fig. 116); thus the two pieces of German silver wire attached to these binding-posts are united. Then arrange the apparatus, as shown in Fig. 116. The lower part of this figure is supposed to join the upper part at the line *AB*. The current is made to run through 15 turns of the galvanoscope. By means of the commutator the current can be easily and quickly reversed through the galvanoscope. Before beginning the experiment, the porous cup should have been soaked through by the acid. This can best be done by pouring the acid into the porous cup a few minutes before the cup is put into the sulphate of copper solution. As the rheostat is now arranged, the current is flowing through 2^m of German silver wire (the resistance of the short piece of copper wire you inserted

between the two binding-posts may be neglected). Read and record the position of the compass needle, which should be placed at the center of the shelf; the coil itself, before the current is allowed to flow through, should be placed nearly in the magnetic meridian. Now, by means of the commutator, reverse the current through

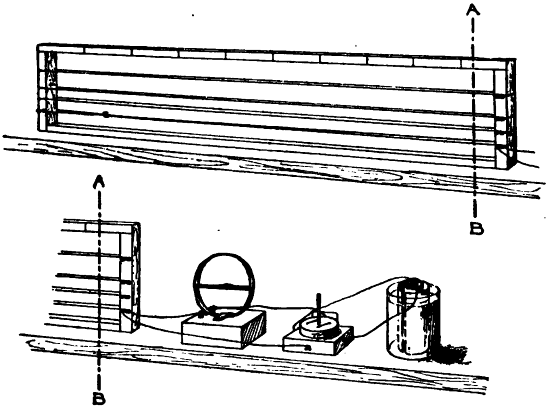


FIG. 116.

the galvanoscope, and again record the reading. Take the average of the two deflections. For example, suppose the N-pointing end of the needle to have come to rest at 42° east of north before the reversal of the current, and at 38° west of north after the reversal, the average, or 40°, should be taken as the true deflection. This method of reversal and averages renders it unnecessary to place the coil *accurately* in the magnetic meridian. (Why?)

Now remove the bit of copper wire which you inserted to connect the two pieces of German silver wire. Connect these same two German silver wires by means of the English binding-post, which should be placed directly

beneath the 90^{cm} mark on the meter stick. The current will now pass through 180^{cm} of the wire.

Read the galvanoscope, which, when you have once begun the experiment, you must not disturb till the experiment is completed. Reverse the current, and read again. Shift the English binding-post to a position under the 80^{cm} mark of the meter stick, so that now the current will flow through 160^{cm} of the wire. Read the galvanoscope; reverse the current, and read the galvanoscope again. By shifting the English binding-post, make the current flow in turn through 140^{cm}, 120^{cm}, 100^{cm}, 80^{cm}, and 60^{cm} of the wire. For every change in position of the English binding-post, read the galvanoscope, reverse the current, and read the galvanoscope again. To make sure that the strength of the cell has not changed during this series of observations, take off the English binding-post and insert, as you did at the start, the bit of copper wire, so that the current may flow through the 2^{m} of the wire. Read and reverse as before. If the strength of the cell is the same as at the start, the average deflection just obtained will be the same as the average deflection obtained at the beginning of the experiment.

A convenient form in which to enter the measurements in your note-book is the following:

cm.	200	180	160	140	120	100	80	60	200
EAST	°	°	°	°	°	°	°	°	°
WEST									
AVERAGE									

From an inspection of your measurements, can you see any indication of what change in the resistance of a wire an alteration in its length makes; that is, does a long wire offer more or less resistance than a short wire of the same thickness?

From the measurements you have made, the exact law stating the dependence of resistance upon the length of wire cannot be inferred. The following is a statement of the law:

The resistance of a wire varies directly as its length.

QUESTION. If there are two wires of the same thickness and material, but one twice as long as the other, how many times as much resistance will the longer wire have than the shorter?

Experiment 144. *To find what influence the area of cross-section of a wire has upon its resistance to the flow of an electric current.*[1]

Apparatus. The same as in the preceding experiment, without the English binding-post.

Directions. Arrange the apparatus as in the preceding experiment, only shift the wires leading from the apparatus to the rheostat from the lower pair of binding-posts to the second pair above, thus putting in continuous circuit with the galvanoscope the 2^m of No. 28 German silver wire. This experiment is a very brief one. Simply read the galvanoscope, reverse the current, read again, and record.

Compare the average deflection thus obtained with the

[1] If this experiment is not performed on the same day as the preceding experiment, it will be necessary to take some of the measurements of that experiment over again to make sure that the cell has undergone no change in strength.

deflections obtained with various lengths of No. 30 German silver wire in the last experiment.

What length of No. 30 is equivalent in resistance to 2^m of No. 28?

The area of cross-section of No. 28 wire is about 1.46 times that of No. 30; so if we call the area of the cross-section of No. 30, 1, that of No. 28 will be 1.46.

Divide the length of No. 28 wire (2^m) by 1.46.

Divide the length of No. 30 wire (of equal resistance) by 1.

Are the two quotients equal or nearly equal?

What relation should you infer exists between the area of cross-section of a wire and its resistance?

157. Divided Circuit. Suppose a branch pipe springs from the side of a main pipe and joins the main pipe again at some distance from the starting-point, the branch thus forming a loop with the main pipe. If water flows along the main pipe, the circuit of water divides when it reaches the point of branching, one part continuing in the main pipe, the other turning aside into the branch, through which it flows till it reunites with the current in the main pipe. Each part of the loop, the main pipe and the branch, offers resistance to the flow of water; but the combined resistance of both parts of the loop is less than that offered by either alone, so that by both branches of the loop taken together less resistance is offered to the flow than by an equal length of the main pipe. In place of a simple loop of two branches we might insert one or more additional branch pipes, and thus have a loop of three branches or more. The current would then divide itself among all

these branches. Just as a current of water divides in branched pipes, so a current of electricity divides in a branched or divided circuit, consisting of two or more wires all springing from the same point and all coming together again at another point. The current of electricity on entering this system of wires at one of these two meeting-points divides itself into smaller currents among the wires, each smaller current flowing along its wire till it is reunited at the other junction with the other smaller currents. The next experiment has for its object the study of resistance in a divided circuit.

Experiment 145. *To find how the resistance of a divided circuit consisting of one loop compares with the resistance of half the loop.*

Apparatus. The same as in the last experiment.

Directions. Connect the two pieces of German silver wire that have already been used on the rheostat by a bit of copper wire as before. Then join this united wire with the other 2^m of No. 30 German silver wire in "multiple arc" circuit; that is, join in divided circuit in such a way that part of the current will go through one of the 2^m pieces of wire, the other part of the current through the other 2^m pieces of wire as shown in Fig. 117.

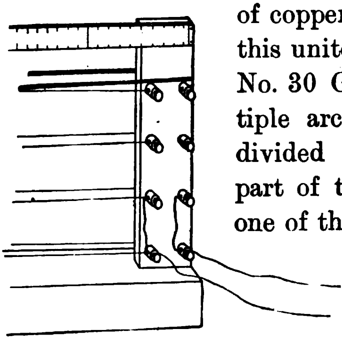


FIG. 117.

Read the galvanoscope, reverse, and read the galvanoscope again; record.

By reference to the record of Exp. 143, what length of

No. 30 German silver wire, as used in that experiment, has a resistance equal to that of the two 2-meter pieces of No. 30 German silver wire joined in multiple arc as in the present experiment?

What part, then, of the resistance of a single 2-meter piece of No. 30 wire is the resistance of two 2-meter pieces of No. 30 wire joined in multiple arc?

158. The Ohm. In order to measure resistances, a unit of resistance, called the *ohm*, has been established.

Definition. *The resistance equal to that offered by a column of mercury of uniform cross-section, at 0°, 106.3^{cm} long and of mass 14.4521^{g}, is called the ohm.*

Copper is a much better conductor of electricity than mercury is. A copper wire whose area of cross-section is $1^{sq\,mm}$ would have to be about 45^{m} long in order to have a resistance of one ohm. Among metals German silver has a high resistance, and its resistance varies only slightly with the temperature. For these reasons German silver wire is employed in the construction of *resistance coils*, which are introduced frequently into electrical circuits to modify the strength of the current.

Experiment 146. *To find what length of German silver wire is equivalent in resistance to 20^{m} of copper wire.*[1]

Apparatus. The same as in the preceding experiment.

Directions. Put the 20^{m} of copper wire, No. 30, into continuous circuit with the galvanoscope. Read the galvanoscope, reverse the circuit, and read again; record.

[1] See note at bottom of page 347.

Compare the average deflection obtained with those obtained in Exp. 143, and so find the length of German silver wire equivalent to that of the copper. Record.

EXAMPLES.

1. If a wire 1^m long has a resistance of 1 ohm, what will be the resistance of a wire of the same material and of the same thickness 2^m long? 10^m long? 0.5^m long?

2. If a wire of $1^{sq\ mm}$ area of cross-section has a resistance of 1 ohm, what will be the resistance of a wire of the same material and length, but of $0.5^{sq\ mm}$ cross-section? $2^{sq\ mm}$ area of cross-section? $10^{sq\ mm}$ area of cross-section?

3. If a wire 1^m long and $1^{sq\ mm}$ area of cross-section has a resistance of 1 ohm, what will be the resistance of a wire of the same material 2^m in length and $2^{sq\ mm}$ in area of cross-section?

4. If a wire 2^m long and 1^{mm} in diameter has a resistance of 3 ohms, what will be the resistance of a wire of the same material 4^m long and 2^{mm} in diameter?

Solution. If a wire 2^m long and 1^{mm} in diameter has a resistance of 3 ohms, a wire 1^m long and 1^{mm} in diameter will have a resistance of 1.5 ohms. If x denotes the resistance in ohms of a piece of wire 4^m long and 2^{mm} in diameter, $\frac{x}{4}$ will denote the resistance of a wire 1^m long and 2^{mm} in diameter. A wire 2^{mm} in diameter has four times the area of cross-section of a wire 1^{mm} in diameter, since the areas of circles are to each other as the squares of their diameters; consequently a wire 1^m long and 1^{mm} in diameter would have a resistance of $\frac{x}{4} \times 4 = x$. But the resistance of a wire 1^m long and 1^{mm} in diameter has previously been found to be 1.5 ohms; hence $x = 1.5.$

That is, the resistance of the wire which is 4^m long and 2^{mm} in diameter is 1.5 ohms.

5. A wire 10^m in length and 1^{mm} in radius has a resistance of 2 ohms; what must be the radius of another wire of the same material whose length is 5^m, in order that it may have the same resistance as the first wire?

6. A wire of length l' and diameter d' has a resistance of r ohms; what is the resistance of a wire of the same material of length l'' and diameter d''?

ELECTRO-MOTIVE FORCE.

159. The Volt. The term *electro-motive force* is one which often occurs in books on the subject of electricity. The term may best be explained for the beginner by analogy, thus: "Just as in the water pipes a *difference of level* produces a *pressure*, and the pressure produces a *flow* as soon as the tap is turned, so *difference of electrical level* produces *electro-motive force*, and electro-motive force sets up a *current* as soon as a circuit is completed for the current to flow through." The unit of E.M.F. (electro-motive force) is called the volt, in honor of Volta, an Italian physicist.

The E.M.F. of the Daniell cell is about 1.1 volts.

It has been proved on investigation that the E.M.F. of a cell depends not upon the size of the plates, but upon the materials comprising the cell.

160. Ohm's Law. Ohm, a German physicist, discovered that the strength of a current of electricity is equal to the E.M.F. driving the current, divided by the resistance which the current encounters; that is,

$$\text{strength of current} = \frac{\text{E.M.F.}}{\text{resistance}}.$$

For the sake of brevity the law is usually stated in the following form:

$$C = \frac{E}{R},$$

where C stands for strength of current, E for E.M.F., and R for resistance.

QUESTIONS. What is the name of the unit of current strength? Of the unit of E.M.F.? Of the unit of resistance?

If there is a circuit in which the E.M.F. is 1 volt, and the resistance 1 ohm, what is the strength of the current?

If a battery having an E.M.F. of 5 volts is placed in a circuit of which the total resistance is 100 ohms, what will be the strength of the current?

BATTERY RESISTANCE.

161. Resistance to the Current in the Cell. The liquids in a cell offer to the flow of the electric current a resistance whose magnitude depends, not only upon the character of the liquids, but also upon the size of the plates and their distance apart.

In writing Ohm's Law in the literal form, it is customary to denote by R the resistance in the part of the circuit that lies outside the battery, and by r the resistance offered by the cell itself; so we have

$$C = \frac{E}{R + r}.$$

R for brevity is called the *external resistance; r* is called the *internal resistance.*

Experiment 147. *To find whether a difference in size and position of the plates of a cell produces a variation in the strength of a current.*

Apparatus. Two Daniell cells with the liquids equally deep in both, one furnished with a plate of zinc and a plate of copper of the same size as in Exp. 140, the other with plates 10^{cm} long and 0.5^{cm} wide, each plate having a wire soldered to it for making electrical connections; a galvanoscope; a rheostat; a commutator.

PART 1. When the cell with large plates is used.

Directions. Before making any measurements, see that the porous cups are thoroughly soaked by the solutions.

Join the cell having the large plates to the commutator; also join the 5-turn section of the galvanoscope to the commutator. By this arrangement the current through the galvanoscope can be readily reversed. Read the galvanoscope, with the plates as far apart in the cell as possible. Then, by means of the commutator, reverse the current, and read the galvanoscope again. Record the readings, and find their average.

PART 2. When the cell with small plates is used.

Directions. Now disconnect the cell from the galvanoscope, and put the cell with small plates into the circuit. Put the plates as far apart as the large plates of the other cell were placed, read the galvanoscope, reverse the current, and read the galvanoscope again. Record the readings, and find their average. In this and subsequent experiments, whenever you are directed to read the galvanoscope, bear in mind that the readings are to be taken one before and the other after the current has been reversed. Put the plates as near together as possible, and read again.

After an inspection of your record, answer the following questions:

With the small cell, do you get the stronger current when the plates are far apart or when they are close together?

Do you get a stronger current with the cell having large plates or with the cell having small plates placed as far apart as those of the large-plate cell are?

NOTE. When the plates in the small-plate cell are put nearer together, the liquid path, which the current in its passage from one plate to the other

traverses, is shortened; consequently, the internal resistance of the cell is diminished. When the plates of two cells, as in the experiment just performed, are of different sizes, but the length of the liquid conductor between them is the same, the cross-section of the liquid conductor in the cell having the larger plates is larger than that of the other cell; hence, the internal resistance of the large-plate cell is less than that of the small-plate cell.

In the experiment just performed the external resistance was small. The next experiment differs only in this respect, that the external resistance is to be made larger.

Experiment 148. *To find whether a difference in size and position of the plates of a cell produces the same variation in the strength of the current when the external resistance is large as it does when this resistance is small.*

Apparatus. The same as in the previous experiment together with a rheostat.

Directions. Put the narrow-plate cell into circuit with the 15 turns of the galvanoscope, and 2^m of the No. 30 German silver wire in the rheostat, inserting the commutator in order to reverse the current through the galvanoscope. With this arrangement repeat Part 2 of the preceding experiment.

Then disconnect the narrow-plate cell, and put in its place the large-plate cell. Then with this arrangement take readings, as in Part 1 of the preceding experiment.

Has the current, in this experiment with the added external resistance, shown more or less fluctuation for change in size and position of the plates than when, as in the preceding experiment, the external resistance was smaller?

Ohm's Law may help you in understanding the results you have obtained.

By inspection of the literal form of Ohm's Law it will be seen, after a little consideration, that when the external resistance, R, is small, a variation in the internal resistance, r, will sensibly affect the value of C; on the other hand, when R is large and r small, a variation in r will produce no sensible change in the value of C.

To make this clearer, let us take a numerical example.

(1) When the external resistance, R, is small.

Let $R = 1$, $r = 2$.

Then $$C = \frac{E}{1+2} = \frac{E}{3}.$$

Change the value of r from 2 to 3.

Then $$C = \frac{E}{1+3} = \frac{E}{4}.$$

Hence, in this case, by increasing r by 1, the value of the current, C, is diminished from $\frac{E}{3}$ to $\frac{E}{4}$, or, expressed in decimal form, from 0.33 E to 0.25 E.

(2) When the external resistance, R, is large.

Let $R = 100$, $r = 2$.

Then $$C = \frac{E}{100+2} = \frac{E}{102}.$$

Change the value of r from 2 to 3.

Then $$C = \frac{E}{100+3} = \frac{E}{103}.$$

Hence, by increasing r by 1, the value of the current, C, is diminished from $\frac{E}{102}$ to $\frac{E}{103}$, or, expressed in decimal form, from 0.0098 E to 0.0097 E.

ARRANGEMENT OF CELLS.

Experiment 149. *To find, provided the external resistance is small, which is the stronger, the current from two cells joined abreast or from two cells joined in series.*

Apparatus. Two Daniell cells provided with full-sized plates (such plates as used in Exp. 147, Part 1); a galvanoscope; a commutator.

PART 1. When the cells are joined abreast.

Directions. Join, so as to make good metallic contact, the ends of the wires leading from the zinc plates of the two cells; also join the wires leading from the coppers of the two cells. Put the cells thus joined into circuit with the 5-turn section of the galvanoscope. Cells joined in this way are said to be joined *abreast.* Insert a commutator into the circuit for reversing the current through the galvanoscope. Read the galvanoscope, and record the deflections.

PART 2. When the cells are joined in series.

Directions. Now disconnect the cells, and join the wire from the zinc of one cell to the copper of the other cell. Put the battery into circuit with the 5-turn section of the galvanoscope; insert the commutator to change the current through the galvanoscope. When arranged in this way, the cells are said to be joined in *series.*

Read the galvanoscope, and record the deflections.

With which arrangement of cells did you obtain the stronger current?

Experiment 150. *To find, provided the external resistance is large, which is the stronger, the current from two cells joined abreast or from two cells joined in series.*

Apparatus. The same as in the last experiment together with a rheostat.

PART 1. When the cells are joined abreast.

Directions. As in Part 1 of the last experiment, join the cells abreast, and then put them in circuit with the 15 coils of the galvanoscope and the 2^m piece of No. 30 German silver wire in the rheostat; insert the commutator to change the current through the galvanoscope. Read the galvanoscope; record its deflections.

PART 2. When the cells are joined in series.

Directions. Now join the cells in series, and put them in circuit with the 15 coils of the galvanoscope, 2^m No. 30 German silver wire, and commutator. Read the galvanoscope; record its deflections.

With which arrangement of cells did you get the stronger current?

162. Arrangement of Cells Abreast and in Series. From your experiments you have been led to the conclusion that when the external resistance is small, the internal resistance must be small; and, on the other hand, when the external resistance is large, the internal resistance must be large.

Careful experiments have shown that the E.M.F. of a battery depends upon the number of cells, joined in *series;* a battery of two cells, joined in series, has an E.M.F. equal to the sum of the E.M.F. of each; if the E.M.F. is the same for each cell, then of course the E.M.F. of the battery is twice that of one of the cells; the E.M.F. of a battery consisting of five cells of the same strength, joined in series, is five times that of a single cell; finally, the

E.M.F. of a battery composed of n cells of equal strength, joined in series, is n times that of a single cell.

It has also been shown that a battery of two cells of equal strength, joined *abreast*, has the same E.M.F. as a single cell; a battery of five cells of equal strength, joined abreast, has the same E.M.F. as a single cell; and, finally, a battery consisting of n cells of equal strength, joined abreast, has the same E.M.F. as a single cell.

Accurate experiments have also shown that the internal resistance of a battery of ten cells joined in *series* is equal to the sum of the internal resistances of the cells; or if the cells are of equal internal resistance, the internal resistance of a battery composed of two cells joined in series is twice that of one cell. In fact, if cells of equal resistance are joined in series, the internal resistance of the battery thus formed is equal to the resistance of one cell multiplied by the number of cells.

If two cells of equal internal resistance be joined abreast, the internal resistance of the battery thus formed would be one-half the resistance of a single cell. If n cells of equal internal resistance be joined abreast, the internal resistance of the group would be $\frac{1}{n}$ of the resistance of a single cell.

Whenever you desire to group a given number of cells in order to form a battery that will give the maximum, or strongest, current through a known external resistance, you should be guided by the following

Rule. *Join the cells in such a way as to make the internal resistance of a battery equal, as nearly as possible, to the external resistance.*

We shall illustrate this rule by the following example:

There are 12 cells, each of E.M.F. 1 volt and internal resistance ½ ohm, and there is a wire the resistance of which is 2 ohms. How must the cells be arranged in order to yield the maximum current through the wire?

Let us arrange the cells in various ways and compute the strength of the current.

(1) All the cells in series (Fig. 118).

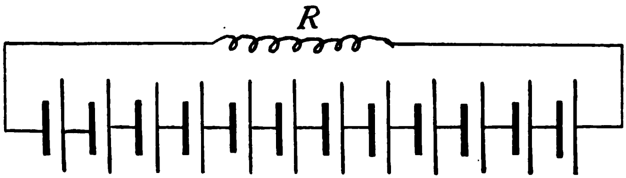


FIG. 118.

By Ohm's Law we can find the strength of the current with this arrangement:

$$C = \frac{E}{R+r} = \frac{12 \times 1}{2 + 12 \times \frac{1}{2}} = \frac{12}{8} = 1.5 \text{ ampères.}$$

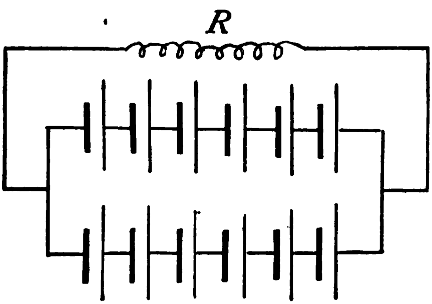


FIG. 119.

(2) 2 cells abreast and 6 in series (Fig. 119).

$$C = \frac{6 \times 1}{2 + \frac{1}{2} \times 6 \times \frac{1}{2}} = \frac{12}{7} = 1.71 \text{ ampères.}$$

(3) 3 cells abreast and 4 in series (Fig. 120).

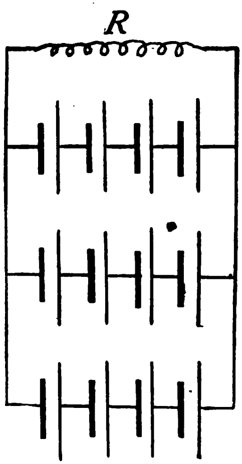


Fig. 120.

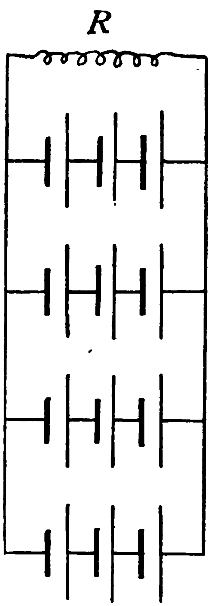


Fig. 121.

$$C = \frac{4 \times 1}{2 + \frac{1}{3} \times 4 \times \frac{1}{2}} = \frac{12}{8} = 1.5 \text{ ampères.}$$

(4) 4 cells abreast and 3 in series (Fig. 121).

$$C = \frac{3 \times 1}{2 + \frac{1}{4} \times 3 \times \frac{1}{2}} = \frac{24}{19} = 1.26 \text{ ampères.}$$

(5) 6 cells abreast and 2 in series (Fig. 122).

$$C = \frac{2 \times 1}{2 + \frac{1}{6} \times 2 \times \frac{1}{2}} = \frac{12}{13} = 0.95 \text{ ampère.}$$

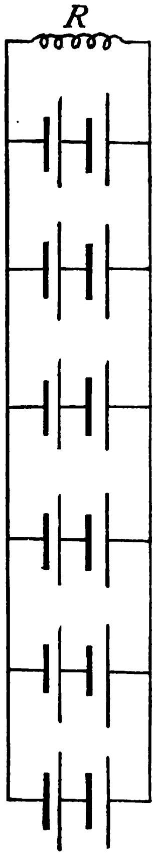


FIG. 122.

(6) All the cells abreast.

$$C = \frac{1}{2 + \frac{1}{12} \times \frac{1}{2}} = \frac{24}{49} = 0.49 \text{ ampère.}$$

We have grouped the cells in many different ways, and find that the strongest current is obtained in the second arrangement, when the internal resistance and the external resistance are as nearly equal as possible. It can be proved by algebra that the strongest current is obtained when the external resistance and the internal resistance are equal; but such a proof is beyond the scope of this book.

EXAMPLES.

1. How should 10 cells, each having an internal resistance of 1 ohm, be arranged in order to send the strongest current possible through a resistance of 12 ohms?

2. How should 12 cells, each having an internal resistance of $\frac{3}{4}$ ohm, be arranged in order to send the strongest current through a resistance of 1 ohm?

Solution. If x denotes the number of cells in series, then $12 \div x$ will denote the number of cells abreast, and the internal resistance of the battery will be $\frac{\frac{3}{4}x}{12 \div x}$; x must have such a value as to make this resistance equal 1, hence

$$\frac{\frac{3}{4}x}{12 \div x} = 1,$$

$$\tfrac{3}{4}x^2 = 12,$$

$$x^2 = 16,$$

$$\therefore x = 4.$$

That is, the battery must be arranged with four cells in series and consequently with three cells abreast.

3. Four cells, each of E.M.F. 1 volt and internal resistance 2 ohms, are to be arranged to send the strongest current possible through an external resistance of 2 ohms; what must the arrangement be?

4. There are two cells, each having a resistance of 2 ohms; how should the cells be arranged in order to get the strongest current when the external resistance is 1 ohm? 2 ohms? 4 ohms?

5. How should 20 Daniell cells, each with a resistance of 1 ohm, be grouped in order to send the strongest current through a resistance of 0.9 ohm?

ELECTRO-MAGNETISM.

163. The Electro-Magnet; Telegraph Key and Sounder. The object of the next experiment will be to bring out the properties of the electro-magnet, and also to show how the electro-magnet can be used to reproduce signals made at a long distance from the electro-magnet.

Experiment 151. *To find what influence a current of electricity has upon a piece of iron, when flowing through a wire wound round the iron.*

Apparatus. A Daniell cell; a cylinder of soft iron about 7^{cm} long and 0.7^{cm} in diameter; insulated copper wire, No. 30 B. & S.; a piece of pine wood about 20^{cm} square; sheet spring brass about 0.5^{mm} thick; a disc of soft iron 0.3^{cm} thick and 0.7^{cm} in diameter.

PART 1. The electro-magnet.

Directions. Wind the wire upon the cylinder of soft iron, just as thread is wound upon a spool, making one or two layers.

When the ends of the wire are joined to the cell, and the disc of soft iron is brought near one end of the iron cylinder, what is the result?

When the circuit is broken, that is, when one end of the wire is disconnected from the cell, what happens to the iron disc?

What is an electro-magnet?

PART 2. The electro-magnet used in the construction of a telegraphic key and sounder.

Directions. Cut from the sheet brass a strip about 10^{cm} long and about 0.7^{cm} wide. Solder the iron disc flatwise to the side of the brass strip near one end (see note on page 139); to the other end solder a flat-headed screw, so that the length of the screw shall be at right angles to the length of the strip, and one edge of the strip shall touch the under side of the head of the screw. Insert the screw into the board at a point on a diagonal about 6^{cm} from the corner, turning the screw till the strip is parallel to the side of the board and the center of the disc is about on a line with the center of the electro-magnet which you have made, when the electro-magnet is laid upon its side on the board. Fasten the electro-magnet in place by a bit of leather laid over the magnet and tacked to the board at each end. Cut another strip from the brass. This strip should be about 12^{cm} long and 1.5^{cm} wide, and should be bent at right angles to its length at a distance of about 3^{cm} from one end, and again at right angles, but in the opposite direction, at a distance of 1^{cm} from the same end. Through the center of this bent end, which is 1^{cm} long, a hole should be made. A screw should be inserted through this hole into the board. Some of the insulating material should be removed from one end of the magnet wire. This end should be wound round the screw, passing

through the brass, two or three times, and then the screw should be "set up" till everything is fast. A screw should be inserted into the board, under the free end of the brass strip, to such a distance as to allow a space of about 1^{cm} between the head of this screw and the under side of the brass strip. When a wire from the cell is joined to this screw, and the other end of the magnet wire, from which the insulating material has been removed, is also connected to the cell, the apparatus is completed. When the brass strip, called the *key*, is pressed down till it touches the screw, the circuit through the electro-magnet is completed, and the little disc of iron, called the *armature*, fastened to the end of the horizontal strip, is attracted to the electro-magnet. The electro-magnet and the armature are together called a *sounder*. Before the sounder will work in a satisfactory manner, the armature may have to be adjusted by turning the screw so as to bring the armature nearer the magnet or to carry it farther away. When the suitable adjustments have been made, the armature will strike the electro-magnet with a sharp click whenever the key is depressed.

Join your instrument to that made by some other student, and try to transmit signals from one instrument to the other.

State how a method might be devised for sending signals from one house to another, by means of an instrument similar to that which you have made.

GALVANOMETERS.

164. The Tangent Galvanometer; the Astatic Galvanometer. The galvanoscope which we have used in

testing the presence of a current of electricity might also be used to measure the strength of the current, the strength of the current being proportional, not to the angle through which the needle is deflected, but to the *tangent*[1] of the angle through which the needle is deflected. In the tangent galvanometer, the current passing through the coil tends to deflect the needle, while the earth's magnetic force resists this tendency; consequently, whenever a very weak current flows through the coil, there is little or no deflection of the needle to be observed. To indicate the presence of a weak current, we must use a more delicate instrument, called an *astatic galvanometer*.

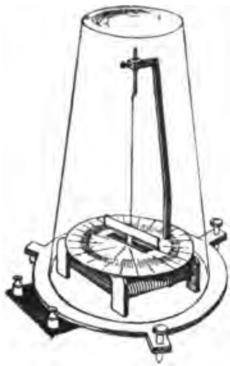


FIG. 123.

This instrument,[2] as shown in Fig. 123, consists of a flat coil of insulated wire, placed in a horizontal position, over which is laid a circular card graduated in degrees. From an upright support is hung, by a delicate fiber, a little contrivance, resembling the letter H turned on its side with the cross line carried beyond one side, as shown in Fig. 124. This H-shaped contrivance is made of aluminum, or some other light and rigid substance. On each of the sides of the H is fastened a magnet, indicated by the heavy black lines in the figure. These magnets are

[1] If from any point in one side of an angle a perpendicular is dropped to the other side, the *tangent* of the angle is equal to the ratio of the perpendicular and that part of the side included between the vertex of the angle and the foot of the perpendicular.

[2] Devised by Dr. E. H. Hall.

turned in opposite directions, so that the N-pointing pole of the upper magnet is directly over the S-pointing pole of the lower magnet, and the magnets are of nearly equal strength. Consequently, the earth's action upon this combination of two magnets has but little effect in making it take a definite direction. In Fig. 124, *E* represents the fiber which supports the magnets, *D* represents the graduated scale over which the side of the H carrying the upper magnet moves, *C* represents the coil through which the current of electricity flows.

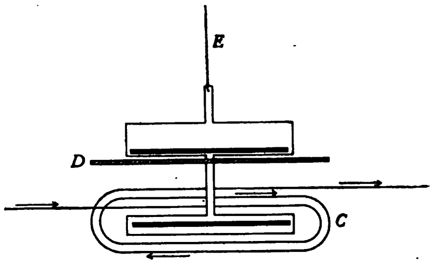


FIG. 124.

The arrows denote the direction in which the current is supposed to be flowing. If the N-pointing pole of the lower magnet is pointing to the right, the action of the current upon this magnet will be to turn its N-pointing pole away from the observer, in consequence of the magnetic action of currents on magnets which you have already examined in Exp. 137. The effect of the current, in the upper half of the coil, on the upper magnet will be to turn it in the direction in which the lower magnet is turned; the effect of the lower half of the coil will be to turn the upper magnet in the opposite direction. This effect, however, is slight, because the lower half of the coil is so far from the magnet. The current acts strongly upon the magnets to turn them in the *same* direction, while the earth's action upon them is very

slight. The H-shaped piece to which the magnets are fastened is very easy to move, since the fiber offers little resistance to twisting. Hence, this arrangement of the magnets and the coil gives an excellent means of detecting the presence of feeble currents.

On the base of the support (Fig. 123) on which the coil rests are leveling screws. By turning these screws, the instrument can be adjusted so that the H-shaped piece will hang freely without touching either the sides of the slot in the cardboard scale or the coil itself. At the upper end of the support to which the end of the fiber is attached is a little screw. By loosening this screw, the magnets can be raised or lowered, thus bringing the upper side of the H-shaped piece the proper distance from the scale. A glass shade covers everything in order to keep out currents of air, which would set the magnets swinging. Two binding-posts in front serve to connect the coil with the wires carrying the current of electricity.

MEASUREMENT OF RESISTANCE.

165. Method of Substitution; Bridge Method. In our experiments on *electrical resistance*, Arts. 151, 152, we made use of the *method of substitution.* We allowed a current of electricity to flow through a wire, and noted the deflection of the galvanoscope needle; then we found another wire which would offer the same resistance to the current which the first wire offered, that is, when a current of the same strength flowed through the second wire as through the first, the needle would be deflected equally. If we had actually known the resistance of the

second wire in ohms, we should also know the resistance of the first wire. To measure resistances by this method, then, we should have to note the deflection produced in the needle of the galvanoscope when a current was flowing through the wire the resistance of which we wish to get; then we should have to *substitute*, for the wire of unknown resistance, wires of known resistance till the current gave the same deflection as before. This method is not as good as the *bridge method*, in which use is made of an instrument called *Wheatstone's bridge*.

Before describing this instrument, we must examine the meaning of the term *equipotential points*.

166. Electrical Potential; Equipotential Points. In order that water may flow from one point to another, there must be a difference of *level*. In order that electricity may flow, there must be a difference of *potential*, or *electrical level*. Whenever two points have the same level, water cannot flow from one to the other; whenever two points have the same potential, a current of electricity cannot flow from one to the other through a wire joining them. Two points which have the same potential are called *equipotential points*. The following experiment will make clearer the term *equipotential points*.

Experiment 152. *To find the relation between the lengths of the segments of a wire divided by a point and the resistances of the segments of another wire divided by a point which has the same potential as the first.*

Apparatus. A frame on which is stretched a German silver wire; a Daniell cell; an astatic galvanometer; a piece of copper wire a

little more than 100^{cm} long; insulated copper wires for making connections.

Directions. Fasten one end of the copper wire and also the end of one wire from the cell in one of the front binding-posts in the frame (Fig. 125). Draw the copper wire straight, and fasten it in the other binding-post together with the end of the other wire from the cell. The current on leaving the wire from the cell enters the brass strip to which the two wires are attached, where it divides, one part flowing along the copper wire, the other

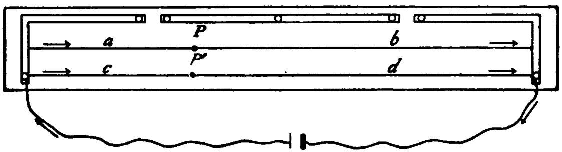


FIG. 125.

part along the German silver wire, as shown by the arrows. These currents unite when they reach the other brass strip to which the wires are attached, and the united currents return to the cell. Connect by a thin wire some point, as P', in the copper wire with the galvanometer, and touch the wire leading from the other binding-post of the galvanometer to the German silver wire at various points till a point is found which has the same potential as the point P', so that no current flows through the galvanometer. Call this point P. Measure and record the lengths, a and b, c and d, of the segments of the two wires. Then change the position of P' and find the corresponding position of P. Record the measurements as before. Again change the position of P'; make measurements and record.

In every case which you have tried, is the following proportion true?

$$a : b = c : d.$$

If you denote the resistances of the segments, *a*, *b*, *c*, and *d*, by *A*, *B*, *C*, and *D*, respectively, knowing that the resistance of a wire is proportional to its length, are the following proportions true?

$$a : b = A : B,$$
$$c : d = C : D.$$

Is the following proportion true?

$$a : b = C : D,$$

$$\text{or } \frac{C}{D} = \frac{a}{b}.$$

167. Theory of Wheatstone's Bridge. Fig. 126 represents a Wheatstone's bridge in the form of a diagram. The horizontal line represents a wire of German silver; the curved line represents another conductor; the wavy lines represent the wires which connect the bridge to the cell; the arrows represent the direction of the current in each conductor; *G* is the galvanometer connected by wires to the equipotential points *R* and *S*. The point *S* has been found by sliding the end of the wire joined to the galvanometer along the German silver wire till no current is indicated by the galvanometer. From the results of the preceding experiment we know that

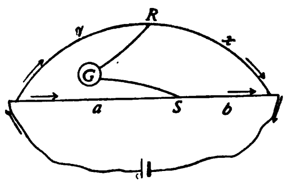


FIG. 126.

$$r : x = a : b,$$

$$\text{or } \frac{x}{r} = \frac{b}{a}. \quad \therefore x = \frac{b}{a} \times r.$$

Experiment 153. *To find, by the bridge method, the resistance of a piece of wire.*

Apparatus. A Wheatstone's bridge (Fig. 127); an astatic galvanometer; a Daniell cell; a box of resistance coils, that is, coils of

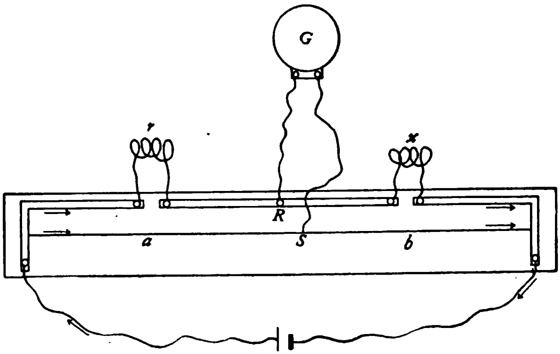


FIG. 127.

wire of known resistance which can readily be put in circuit; a piece of insulated copper wire, No. 30 B. & S., 200^{cm} or 300^{cm} long.

Directions. By means of short, thick, copper wires, whose resistance may be neglected, join the resistance box to the binding-posts on each side of the left-hand gap in the brass strip which nearly surrounds three sides of the instrument, as shown in Fig. 127. Join the resistance to be measured to the binding-posts on either side of the right-hand gap. Connect one wire from the galvanometer to the binding-post at *R*, and the other wire to the binding-post on the piece which slides along the scale over which the German silver wire is stretched. Connect the cell to the binding-posts as shown in the figure. When

everything is ready, touch the sliding part to the German silver wire and quickly remove it, noting in which direction the vane of the galvanometer moves. Then, when the vane has settled somewhat, touch the sliding part again to the German silver wire, but at a different point, and note the direction in which the vane is deflected. The object at first is to get two positions for the sliding part which will deflect the vane in opposite directions. The point on the German silver wire equipotential to R lies somewhere between these two positions, and can now be readily found. If the point S thus found should happen to be near the end of the German silver wire, change the resistance r, by adding resistance or diminishing it, till S is at least 20^{cm} from the end of the wire.

When S has been found so that no current flows through the galvanometer, record the length of a and of b; also record the resistance r. Compute the resistance x.

TEMPERATURE AND RESISTANCE.

168. Temperature Coefficient of Resistance. Whenever a copper wire is heated, its resistance increases. In fact, the resistance of all metals increases with an increase of temperature. The object of the following experiment is to measure the resistance of a copper wire at 0°, and also at 100°, and then to determine what is known as the *temperature coefficient of resistance* for copper.

Experiment 154. *To find by what part of its resistance at 0° a copper wire increases in resistance for a rise in temperature of 1°.*

Apparatus. In addition to that of Exp. 153, a jar of ice-water with many pieces of floating ice; a copper boiler; a Bunsen burner; a thermometer; a glass tube of rather large diameter.

Directions. Wind the wire rather openly round the glass tube, and put the tube thus wound with the wire into the jar of ice-water. Connect the ends of the coil to the bridge by short, thick, copper wires, in the place in which the resistance x (Fig. 127) occupied in Exp. 153. Arrange the other parts of the apparatus as you did in Exp. 153, and find the resistance of the copper wire, after thoroughly stirring the ice and water with the thermometer. Transfer the coil of wire from the ice-water to the copper boiler partly filled with water, and heat this water to boiling. Then find the resistance again of the wire.

From the results which you have obtained, answer the following questions:

(1) What is the resistance of the wire at 0°?

(2) What is the resistance of the wire at 100°?

(3) What is the increase in the resistance of the wire for a rise in temperature of 100°?

(4) What is the average increase in the resistance of the wire for a rise in temperature of 1°?

(5) What part of the resistance at 0° is the increase in resistance for 1°?

Definition. *The temperature coefficient of a conductor is a number which tells by what part of itself the resistance at 0° has increased for a rise in temperature of 1°.*

INDEX.